



# Housing Supply and Housing Bubbles

## Citation

Glaeser, Edward L., Joseph Gyourko, and Albert Saiz. 2008. Housing supply and housing bubbles. *Journal of Urban Economics* 64(2): 198-217.

## Published Version

<http://dx.doi.org/10.1016/j.jue.2008.07.007>

## Permanent link

<http://nrs.harvard.edu/urn-3:HUL.InstRepos:2962640>

## Terms of Use

This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Open Access Policy Articles, as set forth at <http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#OAP>

# Share Your Story

The Harvard community has made this article openly available.  
Please share how this access benefits you. [Submit a story](#).

[Accessibility](#)

# Housing Supply and Housing Bubbles

Edward L. Glaeser  
Harvard University and NBER

and

Joseph Gyourko  
The Wharton School, University of Pennsylvania and NBER

and

Albert Saiz<sup>\*</sup>  
The Wharton School, University of Pennsylvania

Draft of July 16, 2008

## Abstract

Like many other assets, housing prices are quite volatile relative to observable changes in fundamentals. If we are going to understand boom-bust housing cycles, we must incorporate housing supply. In this paper, we present a simple model of housing bubbles which predicts that places with more elastic housing supply have fewer and shorter bubbles, with smaller price increases. However, the welfare consequences of bubbles may actually be higher in more elastic places because those places will overbuild more in response to a bubble. The data show that the price run-ups of the 1980s were almost exclusively experienced in cities where housing supply is more inelastic. More elastic places had slightly larger increases in building during that period. Over the past five years, a modest number of more elastic places also experienced large price booms, but as the model suggests, these booms seem to have been quite short. Prices are already moving back towards construction costs in those areas.

---

<sup>\*</sup> We thank Jose Scheinkman, Andrei Shleifer, participants at the University of British Columbia's 2008 Summer Symposium on Urban Economics and Real Estate, the editors and referee for helpful comments on the paper. Naturally, we remain responsible for any errors. Glaeser thanks the Taubman Center for State and Local Government for financial support. Gyourko and Saiz thank the Research Sponsor Program of the Zell/Lurie Real Estate Center for the same. Scott Kominers, Andrew Moore, and Matt Resseger provided excellent research support.

## **Introduction**

In the 25 years since Shiller (1981) documented that swings in stock prices were extremely high relative to changes in dividends, a growing body of papers has suggested that asset price movements reflect irrational exuberance as well as fundamentals (DeLong et al., 1990; Barberis et al., 2001). A running theme of these papers is that high transactions costs and limits on short-selling make it more likely that prices will diverge from fundamentals. In housing markets, transactions costs are higher and short-selling is more difficult than in almost any other asset market (e.g., Linneman, 1986; Wallace and Meese, 1994; Rosenthal, 1989). Thus, we should not be surprised that the predictability of housing price changes (Case and Shiller, 1989) and seemingly large deviations between housing prices and fundamentals create few opportunities for arbitrage.

The extraordinary nature of the recent boom in housing markets has piqued interest in this issue, with some claiming there was a bubble (e.g., Shiller, 2005). While nonlinearities in the discounting of rents could lead prices to respond sharply to changes in interest rates in particular in certain markets (Himmelberg et al., 2005), it remains difficult to explain the large changes in housing prices over time with changes in incomes, amenities or interest rates (Glaeser and Gyourko, 2006). It certainly is hard to know whether house prices in 1996 were too low or whether values in 2005 were too high, but it is harder still to explain the rapid rise and fall of housing prices with a purely rational model.

However, the asset pricing literature long ago showed how difficult it is to confirm the presence of a bubble (e.g., Flood and Hodrick, 1990). Our focus here is not on developing such a test, but on examining the nature of bubbles, should they exist, in housing markets. House price volatility is worthy of careful study in its own right because it does more than just

transfer large amounts of wealth between homeowners and buyers. Price volatility also impacts the construction of new homes (Topel and Rosen, 1988), which involves the use of real resources that could involve substantial welfare consequences. When housing prices reflect fundamentals, those prices help migrants make appropriate decisions about where to live. If prices, instead, reflect the frothiness of irrational exuberance, then those prices may misdirect the migration decisions that collectively drive urban change.

Most asset bubbles also elicit a supply response, as was the case with the proliferation of new internet firms in Silicon Valley in the late 1990s. Models of housing price volatility that ignore supply miss a fundamental part of the housing market. Not only are changes in housing supply among the more important real consequences of housing price changes, but housing supply seems likely to help shape the course of any housing bubble. We show this is, indeed, the case in Section II of this paper, where we develop a simple model to investigate the interaction between housing bubbles and housing supply.

The model suggests that rational bubbles can exist when the supply of housing is fixed, but not with elastic supply and a finite number of potential homebuyers. We model irrational, exogenous bubbles as a temporary increase in optimism about future prices. Like any demand shock, these bubbles have more of an effect on price and less of an effect on new construction where housing supply is more inelastic. Even though elastic housing supply mutes the price impacts of housing bubbles, the social welfare losses of housing bubbles may be higher in more elastic areas, since there will be more overbuilding during the bubble.

We also endogenize asset bubbles by assuming that home buyers believe that future price growth will resemble past price growth. Supply inelasticity then becomes a crucial determinant of the duration of a bubble. When housing supply is elastic, new construction

quickly comes on line as prices rise, which causes the bubble to quickly unravel. The model predicts that building during a bubble causes post-bubble prices to drop below their pre-bubble levels. The impact of housing supply elasticity on the size of the post-bubble drop is ambiguous because the more elastic places have more construction during a bubble, but their bubbles are shorter.

We then examine data on housing prices, new construction and supply elasticity during periods of price booms and busts. While this empirical analysis is not a test for bubbles, much of the evidence is consistent with the conclusions of our models. For readers who resolutely do not believe in bubbles, the empirical results in the paper still provide information on the nature of housing price volatility across markets with different supply conditions.

In performing the empirical analysis, we distinguish between areas with more or less housing supply elasticity using a new geographical constraint measure developed by Saiz (2008). We also investigate differences in price and quantity behavior between the most recent boom and that which occurred in the 1980s.

During both the 1980s boom and the post-1996 boom, more inelastic places had much larger increases in prices and much smaller increases in new construction. Indeed, during the 1980s, there basically was no housing price boom in the elastic areas of the country. Prices stayed close to housing production costs. If anything, the gap in both price and quantity growth between elastic and inelastic areas was even larger during the post-1996 boom than it was during the 1980s. However, in the years since 1996, there were a number of highly elastic places (e.g., Orlando and Phoenix) that had temporary price explosions despite no visible decline in construction intensity.

The fact that highly elastic places had price booms is one of the strange facts about the recent price explosion. Our model does not suggest that bubbles are impossible in more elastic areas, but it does imply that they will be quite short, and that is what the data indicate. While the average boom in inelastic places lasted for more than four years, the average duration of the boom in more elastic areas was 1.7 years.

Does the housing bust of 1989-1996 offer some guidance for the post-boom years that are ahead of us? During that period, mean reversion was enormous. For every percentage point of growth in a city's housing prices between 1982 and 1989, prices declined by 0.33 percentage points between 1989 and 2006. The level of mean reversion was more severe in more inelastic places, but on average, elasticity was uncorrelated with either price or quantity changes during the bust.

Relatively elastic markets such as Orlando and Phoenix have not experienced sharp increases in prices relative to construction costs in the past, so they have no history of substantial mean reversion. Yet some insight into their future price paths might be provided by the fact that for two decades leading up to 2002-2003, prices in their markets (and other places with elastic supply sides) never varied more than ten to fifteen percent from what we estimate to be minimum profitable production costs (MPPC), which is the sum of physical production costs, land and land assembly costs, and a normal profit for the homebuilder. Those three factors sum to well under \$200,000 in these markets. If these markets return to their historical norm of prices matching these costs, then they will experience further sharp price declines. We now turn to the model that frames our empirical work.

## **II. Housing Bubbles and New Construction: A Simple Framework**

A small, but growing, financial literature has examined the connection between bubbles and asset supply by looking at lock-up provisions (Ofek and Richardson, 2003; Hong et al., 2006). These provisions restrict the ability of asset owners to sell, and when they expire, there can be a burst in the supply of shares on the market and a significant drop in prices. In Hong, Scheinkman and Xiong (2006), the number of truly ebullient buyers is limited, so the extra supply means that the marginal buyer is much more skeptical and prices fall.<sup>1</sup>

Apart from this small but important literature, the bulk of the theoretical work on bubbles written by financial and monetary economists has assumed that the supply of assets is either fixed or determined by a monetary authority. While this assumption is appropriate for some markets, it is not for housing markets, where new homes regularly are built in response to rising prices. In this section, we incorporate housing supply into a simple model of housing bubbles. In doing so, we treat irrational exuberance essentially as an exogenous phenomenon, ignoring recent attempts to micro-found overoptimism (Barberis et al., 2001; Hong et al., 2008). We focus on the extent to which supply mutes or exacerbates housing bubbles and their welfare impacts.

We assume a continuous time model where a city's home prices are determined by the intersection of supply and demand. Demand comes from new homebuyers, whose willingness to pay for housing is based on the utility gains from living in the city and expected housing price appreciation. The supply of homes for sale includes new homes produced by developers and old homes sold by existing homeowners. All homes are physically identical.

---

<sup>1</sup> Shleifer's (1986) work on whether the demand for stocks slopes down can be interpreted as a similar exercise showing that extra demand from index funds for a stock causes prices to rise.

We differentiate between the stock of housing at time  $t$ , which is denoted  $H(t)$ , and the supply of homes for sale at that time period, which is denoted  $S(t)$ . The supply of homes for sale at time  $t$  combines the flow of new construction, denoted  $I(t)$ , and older homes being put on the market by their owners. The supply of new homes is determined by developers, and we assume that the marginal cost of development equals  $c_0 + c_1 I(t)$ . This linear production cost reflects the possibility that there are scarce inputs into housing production, so that the cost of production rises linearly with the amount of production.<sup>2</sup> Ideally, we would model the decision of existing owners to sell as an optimization problem, but this would greatly complicate the model. To simplify, we assume that each current homeowner receives a Poisson-distributed shock with probability  $\lambda$  in each period that forces the homeowner to sell, leave the area and receive zero utility for the rest of time. Homeowners who do not receive the shock do not put their homes on the market. While convenient, not allowing homeowners to time their sale abstracts from potentially important aspects of housing bubbles.<sup>3</sup> Even though each individual homeowner's decision to sell is stochastic, we assume a continuum of homeowners, so the number of existing homes for sale at time  $t$  is deterministic, proportional to the existing stock of homes and equal to  $\lambda H(t)$ .

For the housing market to be in equilibrium, price (denoted  $P(t)$ ) must equal construction costs whenever there is new construction. The level of new homes for sale at time  $t$  therefore equals the maximum of zero and  $(P(t) - c_0)/c_1$ .

Ours is an open city model in which demand comes from a steady flow of new buyers who are interested in moving into the city. At every point in time, there is a distribution of

---

<sup>2</sup> We could also assume that production costs rise with the stock of housing in the area, but this only adds unnecessary complications.

<sup>3</sup> However, Glaeser and Gyourko (2008) conclude the risk aversion limits the ability of consumers to arbitrage prices by delaying the sales of their homes.



risk-neutral buyers who are willing to purchase a home in the city if the expected utility gain from living in the city plus the expected returns when they sell exceeds the price of the house. Consumers have no option to delay purchase; they either buy at time  $t$  or live outside of the city for eternity. We normalize the utility flow outside of the city to equal zero. Inside the city, individual  $i$  receives a time-invariant flow of utility denominated in dollars equal to  $u(i)$ , until that person receives a shock that forces them to sell.

We further assume that, at all points of time, there is a distribution of  $u(i)$  among a fixed number of potential buyers. Some people greatly benefit from living in the city, while others gain less.<sup>4</sup> Across buyers, the utility from living in the city,  $u(i)$ , is distributed uniformly with density  $\frac{1}{v_1}$  on the interval  $[u, v_0]$ . At any point in time, there is a marginal buyer. Individuals whose utility flow from living in the city is greater than that of the marginal buyer will purchase a home, while individuals whose utility flow from living in the city is less than the marginal buyer will not. If the utility flow from the marginal buyer at time  $t$  is denoted  $u^*(t)$ , then the number of buyers will equal  $\frac{v_0 - u^*(t)}{v_1}$ . This represents the level of demand,  $D(t)$ . Alternatively, we can invert the equation and write that the utility flow to the marginal buyer equals  $v_0 - v_1 D(t)$ .

While the utility from living in the area is time invariant, buyers do recognize that at each future date they will sell their homes with probability  $\lambda$  and then receive the sales price of the house. Individuals who bought at time  $t$  understand that with probability  $1 - e^{-\lambda j}$  they will

---

<sup>4</sup> In the financial economics literature, it is less natural to assume that there is a heterogeneous demand for an asset. Instead, heterogeneity is more likely to come from differences in beliefs among investors (Hong et al., 2006).

have sold their house and left the city by time  $t+j$ . This forward-looking aspect of the model means that the current willingness-to-pay for housing will reflect expectations of future prices.

We assume that an individual making decisions at time  $t$  discounts future cash or utility flows at time  $t+j$  with a standard discount factor  $e^{-rj}$ , where  $r$  is the interest rate. The expected utility flow from someone who receives utility  $u(i)$  from living in the city is then given by

$$\frac{u(i)}{r + \lambda} + \lambda E_t \left( \int_{x=t}^{\infty} e^{-(r+\lambda)(x-t)} P(x) dx \right) - P(t), \text{ where } E_t(\cdot) \text{ reflects expectations as of time } t. \text{ If}$$

there is construction, the overall equilibrium in the housing market at time  $t$  satisfies:

$$(1) \frac{v_0 - v_1 D(t)}{r + \lambda} + \lambda E_t \left( \int_{x=t}^{\infty} e^{-(r+\lambda)(x-t)} P(x) dx \right) = P(t) = c_0 + c_1 (S(t) - \lambda H(t)),$$

where  $D(t) = S(t)$ . Equation (1) represents a system of two equations (the consumer's and developer's indifference relations) and two unknowns ( $P(t)$  and  $D(t)$ ). This system can be solved as a function of the current stock of housing  $H(t)$  and expected future prices.

Rational, self-sustaining bubbles can satisfy the demand equation if there is no building. For example, if  $D(t)$  equals a constant,  $D$ , and there is no construction, then the first half of equation (1) will be satisfied at all  $t$ , if  $P(t)$  equals  $\frac{v_0 - v_1 D}{r} + ke^{rt}$ , where  $k$  represents an indeterminate constant of integration. The component  $ke^{rt}$  represents a “rational” bubble, where continuously rising prices justify continuously rising prices. When supply is fixed, stochastic bubbles are also possible.<sup>5</sup>

While rational bubbles are compatible with a fixed housing supply, they cannot exist with elastic housing supply and a finite number of potential homebuyers:

---

<sup>5</sup>There could be a bubble that ends with probability  $\delta$  at each point in time, which survives with probability  $e^{-\delta}$  for each period  $t+j$ , where prices equal  $\frac{v_0 - v_1 D}{r} + ke^{(r+\delta)t}$  during the bubble and then fall to  $\frac{v_0 - v_1 D}{r}$  after the bubble ends.

*Proposition 1:* If housing supply is elastic and the mass of potential buyers at each time  $t$  is finite, then there is no equilibrium where at any time  $t$ ,  $P(t+j) > c_0 + \varepsilon$ , for all positive  $j$  and for any finite  $\varepsilon$ . (All proofs are in the appendix).

Perpetually rising prices means a perpetually rising housing supply, which eventually means there are more homes being offered than there are potential customers in the market. The result in the proposition hinges on our assumption of linear production costs, which implies that there is no cap on the number of homes that could be produced. However, we do not need such an elastic supply of housing to rule out rational bubbles. Rational bubbles cannot exist if the supply of housing, given high enough prices, is greater than the maximum possible number of buyers. Hence, we will turn our attention to irrational bubbles, but before doing so, we discuss the equilibrium without irrationality.

In a perfectly rational world where supply rules out rational bubbles, the value of  $H(t)$  determines whether the city is growing or stagnant. If  $H(t) \geq \frac{v_0 - rc_0}{v_1 \lambda}$ , then prices will be constant at  $\frac{v_0 - v_1 \lambda H(t)}{r}$ , which is less than  $c_0$ , and there will be no construction.

If  $H(t) < \frac{v_0 - rc_0}{v_1 \lambda}$ , then construction will equal  $\gamma \left( \frac{v_0 - rc_0}{v_1 \lambda} - H(t) \right) e^{-\gamma j}$  at each date  $t+j$ , where

$$\gamma = \sqrt{.25r^2 + \frac{\lambda(r + \lambda)v_1}{(r + \lambda)c_1 + v_1}} - .5r.^6 \quad \text{Hereafter, we assume that } H(t) < \frac{v_0 - rc_0}{v_1 \lambda} \text{ so that the}$$

---

<sup>6</sup>The total housing stock,  $H(t+j)$  will equal  $e^{-\gamma j} H(t) + (1 - e^{-\gamma j}) \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} \right)$ , and the amount of housing

on sale at any point in time is given by  $\frac{a_0 + v_0 - rc_0}{v_1} - (\lambda - \gamma) e^{-\gamma j} \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} - H(t) \right)$ .

city is growing. At time  $t$ , the expected price at time  $t+j$  will equal

$$c_0 + c_1 \gamma \left( \frac{a_0 + v_0 - r c_0}{v_1 \lambda} - H(t) \right) e^{-\gamma j}, \text{ and we define this as } P_{R,t}(t+j).$$

### *Irrational Bubbles*

We now turn to less rational bubbles. We model such bubbles as a temporary burst of irrational exuberance by buyers about future prices. We do not require that everyone share this overoptimism, since there is no way of selling housing short in the model. Development occurs instantaneously and reflects current, rather than future, prices so we can remain agnostic about whether developers perceive the bubble in progress. The bubble acts much like any other short-run increase to demand. Prices and construction levels temporarily rise and then fall below equilibrium levels because of the extra housing built during the boom.

We begin by assuming that irrational bubbles represent a completely exogenous burst of overoptimism about future prices that will last for a fixed period of time. Buyers do not know that they are being influenced by a bubble. Equation (1) continues to hold, as long as  $E_t(\cdot)$  is interpreted not as the rational expectation of future prices, but rather the bubble-influenced expectation.

One method of introducing a bubble is to assume that buyers take today's prices and just have some optimistic rule of thumb about the future. We will investigate a model of this form in the next section. In this section, we treat bubbles as an exogenous overestimate of prices relative to the rational expectations equilibrium. Under some circumstances, this exogenous overestimate might be justified by overoptimistic extrapolation, but that will not generally be the case.

If  $P_{R,t}(t+j)$  represents the correct expectation of future prices in the rational expectations equilibrium, then during the bubble people believe that future prices will equal  $P_{R,t}(t+j)$  plus some added quantity. Technically, we assume that during a bubble people overestimate the total value of  $\int_{x=t}^{\infty} e^{-(r+\lambda)(x-t)} P(x) dx$ , relative to the rational expectations equilibrium, by  $\frac{\theta(t)}{r+\lambda}$ . For example, this would be satisfied if people believe future prices will equal  $P_{R,t}(t+j) + \frac{r+\lambda-\phi}{r+\lambda} \theta(t) e^{\phi}$ , but this is only one of many possibilities. The exact nature of buyers' beliefs is not critical for our purpose, as long as they believe that

$$\int_{x=t}^{\infty} e^{-(r+\lambda)(x-t)} P(x) dx \text{ equals } \int_{x=t}^{\infty} e^{-(r+\lambda)(x-t)} P_{R,t}(x) dx + \frac{\theta(t)}{r+\lambda}.$$

This error about future prices will then impact prices at time  $t$ , but the extent of this influence depends on the elasticity of supply. If there were no supply response, then over-optimism will raise prices by  $\lambda$  times  $\frac{\theta(t)}{r+\lambda}$ . As mobility ( $\lambda$ ) increases, the impact of incorrect future beliefs rises because more mobility causes people to care more about the future. Allowing a construction response will mute the price impact of overoptimism because added supply means that the marginal buyer has a lower utility benefit from living in the city. In a growing city that experiences a bubble, prices at time  $t$  will equal the rational expectations price,  $c_0 + c_1 \gamma \left( \frac{a_0 + v_0 - r c_0}{v_1 \lambda} - H(t) \right)$ , plus  $\frac{\lambda c_1 \theta(t)}{(r+\lambda) c_1 + v_1}$ . The price impact of the bubble increases with mobility and the inelasticity of supply ( $c_1$ ), and decreases with the heterogeneity of preferences ( $v_1$ ).

Our next two propositions consider the impact of an exogenous bubble where  $\theta(t) = \bar{\theta}$  for the period  $t$  to  $t+k$  and then equals zero thereafter. Proposition 2 describes the general impact of the bubble on prices, investment and the housing stock. Proposition 3 examines the interaction between the bubble and housing supply elasticity. Recall that we assume

$$H(t) < \frac{v_0 - rc_0}{v_1 \lambda}, \text{ so the city is growing:}$$

*Proposition 2:* During the bubble, an increase in  $\bar{\theta}$  causes prices, investment and the housing stock to increase. The impact on prices and investment becomes weaker over the course of the bubble, but the impact on the housing stock increases over the course of the bubble. When the bubble is over, higher values of  $\bar{\theta}$  decrease housing prices and investment, and increase the housing stock.

During the period of the bubble (or any demand boost), there is a surge in prices and new housing construction. Since the bubble's magnitude is fixed during its duration, the price and development impact of the bubble actually declines over time as new housing is built, but this implication will disappear below when we assume that beliefs are fueled by the experience of price growth.

When the bubble ends, housing prices fall below what they would have been if the bubble had not happened. The price crash represents both the end of the overoptimism and the extra supply that was built during the high price bubble period. The extra supply of housing results in lower prices and construction levels *ex post*. Unsurprisingly, longer bubbles have a larger impact. Our primary interest, however, is in the interaction between the bubble and housing supply, which is the subject of Proposition 3:

*Proposition 3:* (a) During a bubble, higher values of  $c_1$  -- a more inelastic housing supply-- will increase the impact of  $\bar{\theta}$  on prices. Higher values of  $c_1$  will decrease the impact that  $\bar{\theta}$  has on total housing investment during the boom.

(b) After the bubble is over, higher values of  $c_1$  will reduce the adverse impact that the bubble has on price levels when  $c_1$  is sufficiently high, and will increase the impact that the bubble has on prices when  $c_1$  is sufficiently low. If the bubble is sufficiently long, then higher values of  $c_1$  increase the negative impact that the bubble has on price levels once it has burst.

Part (a) of Proposition 3 details the impact that more inelastic housing supply (reflected in higher values of  $c_1$ ) has on the bubble during its duration. When housing supply is more inelastic, the bubble will have a larger impact on price and a smaller impact on the housing stock. The impact of  $c_1$  on the flow of investment depends on the stage of the bubble. Early in the bubble, places with more elastic housing supply will see more new construction in response to the bubble. Over time, however, the housing investment response to the bubble will actually be lower in places where supply is more elastic because there has already been so much new building in those markets.

The interaction between an exogenous bubble and supply inelasticity is quite similar to the connection between elasticity and any exogenous shift in demand. During the time the bubble is raging, more inelastic places will have a larger shift in prices and will also experience a sharper reduction in prices when the bubble ends. More elastic places will have a larger increase in new construction, at least during the early stages of the bubble, and will always experience a larger increase in total housing stock from the bubble.<sup>7</sup>

Part (b) of the proposition discusses the impact of housing elasticity on prices when the bubble is over. The post-bubble impact on prices can either be larger or smaller in places with more inelastic housing supply because there are two effects that go in opposite directions.

---

<sup>7</sup> One of the counter-factual elements in this model is that prices and investment drop suddenly at the end of the bubble rather than declining gradually, as they generally do in the real world. The price drop at the end of the bubble will be larger in more inelastic places with higher values of  $c_1$ . Higher values of  $c_1$  moderate the decline in new construction at the moment when the bubble ends.

First, places with more elastic housing build more homes in response to the bubble. These extra homes act to reduce price levels when the bubble is over. Second, the impact of extra homes on prices will be lower in places where housing supply is more elastic. Either effect can dominate, so it is unclear whether the price hangover from a housing bubble is more severe in places where supply is more or less elastic.

Just as more elastic housing supply can either increase or decrease the impact of the bubble on *ex post* prices, more elastic housing supply can either increase or decrease the welfare consequences of a bubble. Abstracting from the impacts that bubbles might have on savings decisions or on risk averse consumers, the inefficiency of a housing bubble comes from the misallocation of real resources—that is, the overbuilding of an area. This overbuilding will be more severe in places where housing supply is more elastic.

We can calculate the expected welfare loss for a bubble that occurs only at period  $t$ , so that buyers during that time period overestimate future prices, but no one after them makes that same mistake. In that case, the number of people who buy housing at period  $t$  equals

$$\lambda H(t) + \gamma \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} - H(t) \right) + \frac{\lambda \theta(t)}{(r + \lambda)c_1 + v_1}, \text{ which equals the share of people who would}$$

have bought housing absent the bubble plus  $\frac{\lambda \theta(t)}{(r + \lambda)c_1 + v_1}$ . The total welfare loss is found by

adding consumer welfare for the consumers who bought during that time period minus

development costs, or  $-\frac{\lambda^2 \theta(t)^2}{(r + \lambda)c_1 + v_1}$ . This is decreasing in  $c_1$ . More inelastic markets will

have bigger price swings in response to a bubble, but the welfare losses will be smaller, since there is less of a construction and mobility response.



The parallel of this in the real world is that if there is a housing bubble in a very inelastic market such as greater Boston, prices may swing a lot, but there will be little change in migration or construction patterns. If there is a housing bubble in a more elastic area such as Phoenix and Orlando, then many thousands of extra homes will be built and many people will make migration mistakes.

### *Self-Reinforcing Bubbles and Adaptive Expectations*

The conclusion that greater resource misallocation results from overbuilding in more elastic places assumes that the magnitude of the bubble is independent of housing elasticity. However, if bubbles are endogenous and sustained by rising prices, then more inelastic places might have bigger and longer bubbles, because rising demand is more likely to translate into rising prices in those markets. To examine this possibility, we assume that bubbles take the form of an irrational belief that prices will continue to rise at a fixed rate for all eternity.

We continue to work with our continuous time model, but we now assume that individuals update their beliefs only at discrete intervals. For example, if the bubble begins at time period  $t$ , then between period  $t$  and period  $t+1$ , individuals have exogenously come to the view that future prices will increase perpetually so that buyers believe that  $P(t+j)$  will equal  $P(t) + \varepsilon \cdot j$  for all  $j$ . While this hybrid discrete-continuous time set-up is somewhat unusual, it is inspired by a financial econometrics literature examining settings where researchers only observe a continuous time process at discrete intervals (Hansen and Scheinkman, 1995).

At time period  $t+1$ , individuals update their beliefs and base them on price growth between period  $t$  and  $t+1$ . Between time periods  $t+1$  and  $t+2$ , people extrapolate from price growth between period  $t$  and  $t+1$  and believe that  $P(t+j)$  will equal

$P(t+1) + (P(t+2) - P(t+1)) \bullet (j-1)$  for all  $j$ . At every subsequent period  $t+x$ , where  $x$  is an integer, updating again occurs. As long as prices continue to rise between period  $t+x$  and  $t+x+1$ , buyers believe that future prices,  $P(t+j)$ , will equal

$P(t+x) + (P(t+x) - P(t+x-1)) \bullet (j-x)$ .<sup>8</sup> The future growth in price is assumed to equal the growth rate during the period between the last two integer time periods. We let  $g(t)$  denote the expected future growth rate at time  $t$ .

As soon as price declines for one period, beliefs revert to rationality and the equilibrium returns to the rational expectations equilibrium with, of course, an extra supply of housing. If prices grow by less than they did the period before, prices will start to fall, and we will refer to this as the end of the bubble.

We will normalize the date of the start of the bubble to be zero and assume that the city has reached its long-run stable population level at that point, so that  $H(0) = \frac{v_0 - rc_0}{v_1 \lambda}$  and

$P(0) = c_0$ . Because more housing will be built with the boom, if the irrational expectations disappear, the long-run steady state is that new construction will then end forever and  $H(t)$  will remain at its highest level at the peak of the boom. Prices will then satisfy

$P(t) = \frac{v_0 - v_1 \lambda H(t)}{r}$  for the rest of time. We let  $H_I(t) = H(t) - H(0)$  denote the incremental

housing built during the boom, so that  $P(t) = c_0 - \frac{v_1 \lambda H_I(t)}{r}$  after the bubble ends. New

housing built during the boom has a permanent, negative impact on prices after the boom is over, since this city begins at steady-state population levels, but the impact of the amount of housing built is independent of housing supply elasticity in the area.

---

<sup>8</sup> See Mayer and Sinai (2007) for a recent examination of the possible role of adaptive expectations in the recent boom.

The demand side of the model is captured by the equation:

$$(1') \quad P(t) = \frac{v_0 - v_1 D(t)}{r + \lambda} + \lambda E_t \left( \int_{x=t}^{\infty} e^{-(r+\lambda)(x-t)} (P(t) + g(t) \bullet (x-t)) dj \right) = \frac{v_0 - v_1 D(t)}{r} + \frac{\lambda g(t)}{r(r + \lambda)},$$

which must hold throughout the bubble. The supply side continues to satisfy

$$c_0 + c_1 I(t) = P(t), \text{ which implies that } I(t) = \frac{\lambda g(t)/(r + \lambda) - \lambda v_1 H_I(t)}{rc_1 + v_1}.$$

response into the pricing equation implies that 
$$P(t) = c_0 + \frac{\lambda c_1}{rc_1 + v_1} \left( \frac{g(t)}{r + \lambda} - v_1 H_I(t) \right).$$

Unsurprisingly, supply mutes the price response to optimistic beliefs.

During the first period, individuals believe that  $g(t) = \varepsilon$ . In the second period,  $g(t)$  reflects the price growth during the first period. In the third period,  $g(t)$  reflects the price growth during the second period. For the bubble to continue, not only must prices rise, but they must rise at an increasing rate. Without ever-increasing optimism, new construction would cause prices to fall. Yet, the ever-rising prices that are needed to fuel increasingly optimistic beliefs are only possible if housing supply is sufficiently inelastic and if people are sufficiently patient:

*Proposition 4:* If  $\lambda < \frac{r^2}{1-r}$ , then no bubble can persist beyond the first time period. If  $\lambda > \frac{r^2}{1-r}$ , then at time one, there exists a  $c_1$ , denoted  $c_1^*$ , where an initial bubble will persist if and only if  $c_1 > c_1^*$ . If  $\lambda < \frac{2r^2}{1-r}$ , then no bubble can persist beyond the second time period, but if  $\lambda > \frac{2r^2}{1-r}$ , then there exists a second value of  $c_1$ , denoted  $c_1^{**}$  which is greater than  $c_1^*$ , where an initial bubble can persist if and only if  $c_1 > c_1^{**}$ .

Proposition 4 shows the connection between housing inelasticity and endogenous bubbles. If housing supply is sufficiently elastic, then a temporary burst of irrational exuberance will be met with an abundant supply of new housing, which will in turn ensure

that housing price gains are less than those expected by the optimistic buyers. The failure of reality to match expectations will quickly cause the bubble to pop. However, if housing supply is inelastic, then prices will increase by more and these early, observed price increases cause the bubble to persist, because we have assumed adaptive expectations.

The degree of inelasticity needed for the bubble to survive increases over time, so that only places with extremely inelastic housing supplies can have long-lasting bubbles. These effects mute the implication of the last section that housing bubbles will cause greater welfare losses through overbuilding in more elastic areas since the construction response to overoptimism is more severe. The adaptive expectations model essentially endogenizes the degree of overoptimism (after the initial perturbation) and finds that self-sustaining overoptimism is more likely in inelastic areas. This means that the overall social costs of bubbles may well be larger in places where housing is more inelastically supplied. The connection between inelasticity and the housing produced by a bubble is explored in Proposition 5:

*Proposition 5:* If  $c_1 < c_1^*$ , so that the bubble lasts exactly one period, then the amount of excess housing built over the course of the bubble will be decreasing monotonically with  $c_1$ , and the price loss at the end of the bubble is monotonically increasing with  $c_1$ . If  $c_1^* < c_1 < c_1^{**}$ , so that the bubble lasts exactly two periods, then housing built during the bubble and the price drop at the end of the bubble may either rise or fall with  $c_1$ .

The first part of Proposition 5 shows that if the bubble lasts exactly one period, then places with more inelastic housing supply will have less housing built during the bubble. This corresponds to the results of the last section, since a one-period bubble is essentially a wholly exogenous bubble. In the case of a one-period bubble, the price drop at the end of the bubble will also be higher in more inelastic areas, just as we found in the previous section.

The second result in the proposition shows that any clear implications disappear when the bubble last for two periods. In that case, the degree of overoptimism in the second period will be increasing with housing supply inelasticity since the price increase during the first period is also increasing with inelasticity.

The core inequality that determines whether new construction rises with inelasticity is

$$(2) \quad r \left( r + \lambda + \frac{\lambda c_1}{(rc_1 + v_1)} \right) \left( 2e^{\frac{2\lambda v_1}{rc_1 + v_1}} - 1 \right) > 1 - e^{\frac{\lambda v_1}{rc_1 + v_1}}.$$

This obviously fails when  $r$  is sufficiently small. When  $v_1$  is sufficiently small, the inequality must hold. More inelastic housing actually leads to more new construction over the course of the bubble in this case because more inelastic housing in the first period causes the price appreciation during that period to be higher, which in turn pushes up new construction during the second period. This finding depends on our adaptive expectations assumption. We intend this result to be interpreted as showing only the possibility that inelasticity can increase housing supply, not to claim generality for it.

The ambiguous effect of supply inelasticity on new construction is mirrored in the ambiguous effect of supply inelasticity on price declines at the end of one period. More inelastic housing supply means higher prices during the second period of the bubble, but it also means less new housing supply. These effects work in opposite direction in their impact on the price drop after the bubble, so the overall impact of inelasticity on price declines is ambiguous.

In sum, the model has two clear empirical predictions and is ambiguous on a number of other points. During a bubble, price increases will be larger in more inelastic areas, and bubbles will be shorter and rarer in more elastic areas. The implications about construction

inelasticity and housing production during the bubble are ambiguous, because more inelastic places have bigger bubbles but smaller construction responses during the bubble. The impact of elasticity on welfare and on price and post-bubble price and investment declines also are ambiguous.

### **III. Housing Market Data and Empirical Analysis**

Our empirical work is meant to examine the implications that the model has about the interaction between overoptimism and housing supply. However, in no sense do we attempt to test for the existence of bubbles or any other form of market irrationality. We begin with a description of our data and follow with an examination of how markets behaved across metropolitan areas facing different degrees of constraints to supply. We also study three distinct periods of time, which are defined by the peaks and troughs in national price data.

#### *Data Description and Summary Statistics*

The Office of Federal Housing Enterprise Oversight (OFHEO) indices of metropolitan area-level prices are the foundation of the home price data we use.<sup>9</sup> To create price levels, we begin by computing the price of the typical home as the weighted average of the median house value from the 2000 census across each county in the metropolitan area, where the weights are the proportion of households in the county. That value is then scaled by the relevant change in the OFHEO index over time. This then proxies for the price over time of the median quality home in 2000 in each market. All prices are in constant 2007 dollars, unless explicitly noted.

---

<sup>9</sup> The OFHEO series can be downloaded at the following URL: [http://www.ofheo.gov/hpi\\_download.aspx](http://www.ofheo.gov/hpi_download.aspx).

Housing permits serve as our measure of new quantity. Permits are available at the county level from the U.S. Bureau of the Census.<sup>10</sup> We aggregate across counties to create metropolitan area-level aggregates using 2003 definitions provided by the census.

While the price and quantity series are available on a quarterly basis, we use annual data because our focus is on patterns over a cycle, not on higher frequency, seasonal effects. Annual data arguably are less noisy, too. Moreover, when we want to compare prices to costs, we must use construction cost data that is only available annually. All construction cost data are from the R.S. Means Company (2008). We use their series for a modest quality, single family home that meets all building code requirements in each area.<sup>11</sup>

These construction cost data only capture the cost of putting up the improvements. The total price of a home also should reflect land and land assembly costs, plus a profit for the builder. What we term the minimum profitable production cost (MPPC) in each metropolitan area is computed as the sum of the following three components: (a) physical construction costs (CC), which represent the cost of putting up the improvements; (b) land and land assembly costs; and (c) the gross margin needed to provide normal profits for the homebuilder entrepreneur.

We calculate constructions costs, using the R.S. Means data for an 1,800ft<sup>2</sup> economy-quality home. Land costs are much less readily available, as there is very little data on residential land sales. Hence, we assume that these costs amount to one-fifth of the total value of the house (land plus improvements). This assumption seems likely to be conservative in the sense that true land costs probably are lower in many of the Sunbelt markets and exurbs, but it

---

<sup>10</sup> Permit data are available at the following URL: <http://censtats.census.gov/bldg/bldgprmt.shtml>.

<sup>11</sup> This series is for an ‘economy’ quality home. It is the most modest of four different qualities of homes whose construction costs the R.S. Means Company tracks over time. We have used these data in previous work. Glaeser and Gyourko (2003, 2005) report more detail on this series.

certainly understates land value in many urban cores and in the constrained markets on both coasts. We assume that the typical gross profit margin for a homebuilder is 17%, as this yields net profits of 9%-11%, depending upon the overhead structure of the firm. Thus, minimum profitable production cost (MPPC) is computed as  $MPPC = [(Construction\ Cost\ per\ Square\ Foot * 1,800) / 0.8] * 1.17$ .<sup>12</sup>

Our primary measure of supply side conditions is taken from Saiz (2008). Using GIS software and satellite imagery, Saiz (2008) measured the slope and elevation of every 90 square meter parcel of land within 50 kilometers of the centroid of each metropolitan area. Because it is very difficult to build on land with a slope of 15 degrees or more, we use the share of land within the 50 kilometer radius area that has a slope of less than 15 degrees as our proxy for elasticity conditions. Saiz (2008) first determines the area lost to oceans, the Gulf of Mexico, and the Great Lakes using data from the United States Geographic Service (USGS). Land area lost to rivers and lakes is not excluded in these calculations. The USGS' Digital Elevation Model then is used to determine the fraction of the remaining land area that is less steeply sloped than 15 degrees. This measure of buildable land is based solely on the physical geography of each metropolitan area, and so is independent of market conditions.<sup>13</sup>

Given the literature on endogenous zoning, the more plausibly exogenous nature of Saiz's physical constraint measure leads us to favor it over the other primary supply-side proxy we experimented with, the Wharton Residential Land Use Regulatory Index (WRLURI)

---

<sup>12</sup> The greatest measurement error in this rough estimate of the minimum cost at which housing could be produced almost certainly arises from our underlying land value assumption. Data on residential land prices across markets is very scarce in the cross section and is unavailable over time. Our assumption is based on a rule of thumb used in the building industry, but conditions do vary substantially across markets as noted just above, and they also could have over time. We discuss this issue more below when this variable is used in our empirical analysis.

<sup>13</sup> The use of geography in this context follows in the tradition of Hoxby (2000). In the urban field, see also Duranton, et. al. (2008), Holmes and Lee (2008) and Strange and Rosenthal (2008).



from Gyourko, Saiz and Summers (2008).<sup>14</sup> This is a measure of the strictness of the local regulatory environment based on results from a 2005 survey of over 2,000 localities across the country. It was created as an index using factor analysis and is standardized to have a mean of zero and a standard deviation of one, with a higher value of the index connoting a more restrictive regulatory environment. While we prefer the geographic constraint for our purposes, the two variables are highly (negatively) correlated, so that a city with a smaller share of developable land tends to have a higher Wharton index value.<sup>15</sup>

We work with annual house price and quantity data from 1982-2007, with 1982 being the first year we are able to reliably match series across our metropolitan areas. Table 1 reports recent values on these variables. The first column provides 2007 prices of the median quality home (as of 2000) in each metropolitan area. As expected, there is a wide range of price conditions across markets. The mean value is \$234,168, with a relatively large standard deviation of \$141,822. The price at the top of the interquartile range is double that at the bottom (\$275,101 versus \$134,276), but even that does not capture the extreme values in the tails. The metropolitan area in the 10<sup>th</sup> percentile of the distribution had a home price of \$116,205 versus \$447,962 for the area in the 90<sup>th</sup> percentile. The minimum price is \$103,013, and the maximum price is \$721,694.

---

<sup>14</sup> The literature on whether zoning is endogenous is a lengthy one. See McDonald and McMillan (1991) and Wallace (1988) for two early influential pieces on this topic.

<sup>15</sup> The simple correlation between the index and the geographic constraint variables across the 78 metropolitan areas for which we have at least one community response to the Wharton survey is -0.40. For metropolitan areas with multiple communities responding to the survey, we use the mean index value across those communities when computing this correlation. Given the strong correlation, it is not very surprising that our regression results reported below are qualitatively similar whether we use the geography- or regulation-based variable below. [The signs are reversed, of course.] Those results are available upon request, and the underlying micro data from the Wharton survey can be accessed at [http://real.wharton.upenn.edu/~gyourko/Wharton\\_residential\\_land\\_use\\_reg.htm](http://real.wharton.upenn.edu/~gyourko/Wharton_residential_land_use_reg.htm). We also examined data from Saks (2008), which created another index of supply-side restrictiveness based on other previous research mostly surveys. Her series also is strongly correlated with Saiz (2008) and Gyourko, Saiz and Summers (2008), but it covers many fewer metropolitan areas (about 40).

Minimum profitable production costs, also as of 2007, are reported in the second column of Table 1. There are only five metropolitan areas for which this value exceeds \$200,000, and the maximum value is only \$224,641.<sup>16</sup> The mean is \$166,386 and the standard deviation is a relatively small \$20,032, so production costs do not vary across areas nearly as much as prices do. Obviously, there are many markets in which prices are far above our calculated MPPC values.

Figure 1 illustrates some of the time series variation in these data in its plot of the average ratio of price-to-MPPC across the 79 metropolitan areas in our sample. This plot indicates that the prices were significantly below our estimate of minimum profitable production costs for much of the 1980s. Indeed, it was only during the recent boom that average prices rose above construction costs.

Because the OFHEO-based repeat sales index measures appreciation on existing homes that sell, while our estimate of production costs is for a new house, the comparison is of prices on older homes with the costs of new product. Hence, it is not so surprising to see prices below our estimate of MPPC. While this bias is unavoidable in any comparison of new home costs to those based on repeat sales indexes, it does suggest that any positive gap between prices and production costs is a conservative measure of that difference. That a large gap clearly developed between 1996-2005 suggests that the recent boom may have been different in certain respects.<sup>17</sup>

---

<sup>16</sup> The New York metropolitan area has the highest construction costs. The four other areas with costs above \$200,000 are in the Boston and San Francisco Bay areas.

<sup>17</sup> Obviously, we can mechanically have the price-to-cost ratio be near one in the early 1980s if we were to assume that land constituted a lesser than 20 percent share of overall property value. But, as just noted, there is no reason to believe that this ratio could not ever be less than one. Our focus here is on the time series pattern, and we do not believe there is any reasonable chance the upward slope could be explained by errors in our underlying assumptions for the MPPC.

With respect to the supply side, there is considerable dispersion in Saiz's series on the fraction of developable land area. The sample mean for this variable is 81.6%, with a standard deviation of 19.8%. The 10<sup>th</sup> percentile value is only 51%, compared to a 90<sup>th</sup> percentile value of 100% developable land. The most physically constrained metropolitan area is Oxnard-Thousand Oaks-Ventura (20.7% developable land), while 13 markets are completely unconstrained by this measure (e.g., Dayton, OH, and Wichita, KS, which are off the coasts and have no steeply sloped land parcels within 50 kilometers of their centroids).

To gain initial insight into how house prices and quantities vary with supply elasticity conditions over our full sample period, we estimated specifications (3) and (4) below, which allow us to see whether national price and quantity cycles generate differential local impacts depending upon local supply elasticity conditions. After controlling for metropolitan area and time (year) fixed effects in the price and quantity specifications, the local effects can be seen in the coefficients on the interaction of the developable land share variable in each market ( $DevShr_i$ ) with the (log) average annual national house price across the 79 areas in our sample ( $Price_{US,t}$ ). Specifically, we estimate

$$(3) \log Price_{i,t} = \alpha + \beta * Year_t + \gamma * MSA_i + \delta * (DevShr_i * \log[Price_{US,t}]) + \varepsilon_{i,t}$$

$$(4) \log Permits_{i,t} = \alpha' + \beta' * Year_t + \gamma' * MSA_i + \delta' * (DevShr_i * \log[Price_{US,t}]) + \varepsilon'_{i,t},$$

where  $i$  indexes the metropolitan areas,  $t$  represents each year from 1982-2007, and  $\varepsilon$  is the standard error term.

The coefficients  $\delta$  and  $\delta'$  indicate how price and quantity vary with the degree of supply side elasticity, controlling for the state of the cycle (with average prices), as well as general year and market effects. The results from equation (3) yield a negative value for  $\delta$  (of

-0.0228; std. err. = 0.0009), which implies that prices move much less over the national cycle in more elastic markets. The results from equation (4) yield  $\delta' = 0.0124$  (std. err. = 0.0028), indicating that permits move much more over the national cycle in more elastic places.<sup>18</sup> These basic results both suggest that our geographic constraint variable is proxying for supply side conditions as we had intended and that price and quantity responses vary as our model predicts. We now turn from these summary statistics to analyses of specific boom and bust periods in the housing market.

#### **IV. The 1982-1996 Cycle**

The bulk of variation in housing prices is local, not national, but there still are periods when there is enough co-movement across areas that there are recognizable national housing cycles. Between 1982 and 1989, three-quarters of our metropolitan areas had nominal price growth greater than 21% over the eight year period. Inflation was relatively high during the 1980s, but real appreciation throughout the sample still averaged 14.6% over this time span.

As Figure 2 shows, 1989 was the peak of average housing prices in our sample between 1982 and 1996, so it is the natural peak of a housing cycle. Averaged across all of our markets, prices only started to rise consistently again in 1997, so we will define the 1989-1996 period as the housing bust on the other side of this cycle. Over this latter time period, half of our markets (40 out of 79) experienced real price declines, although barely 10 percent (10 out of 79) experienced nominal price drops. Table 2 reports more detail on the distribution of real and nominal price growth over time.<sup>19</sup>

---

<sup>18</sup> Each specification is estimated using 2,212 observations (79 metro areas over 28 year). For the price equation, the  $R^2 = 0.91$ ; for the quantity equation, the  $R^2 = 0.85$ .

<sup>19</sup> Even over this national boom-bust cycle, the bulk of year-to-year variation in price changes was local. For example, if we regress annual percentage real price growth in each market on year dummies, we find that those

Accompanying the boom and bust of housing prices is an analogous change in housing production. Figure 3 depicts the movement in new residential permits issue over time. The plot is of the yearly sum of all housing permits issued in our 79 market sample. This quantity series shows the same basic rise, decline and rise as the price series, although permits seem to peak and trough earlier than prices. Perhaps this is because builders are able forecast price changes, but that is an issue for future research.

Was the boom-bust cycle in housing prices a bubble or a response to fundamentals? The national economy was doing reasonably well in the mid- to late 1980s, and interest rates and inflation had fallen from their dramatic heights during the early 1980s. While there were observers in the 1980s who called the price gains a bubble, the fact that real conditions were changing dramatically made any such claims highly debatable. The best case for a bubble, at least in some markets, was that prices fell so dramatically over a short period of time after the boom without any obviously comparable changes in fundamentals.

For example, real prices in the Los Angeles-Long Beach-Glendale metropolitan area rose by 67 percent between 1984 and 1989. Over the next five years, real values then declined 33 percent. We understand those who find it hard to look at Los Angeles' prices in both 1989 and 1994 and to think that these dramatic changes in prices were actually driven by changes in real features of southern California.<sup>20</sup> In retrospect, it is just as possible to take the view that 1994 was unduly pessimistic as it is to take the view that 1989 was unduly optimistic. In either case, something seems to have been amiss with standard rational pricing models, as discussed in the Introduction.

---

dummies explain only 14% of the variation in housing price appreciation. Still, this implies there is a meaningful amount of national variation, possibly driven by changes in interest rates, the national economy, demographics and perhaps even nationwide beliefs about the future of the housing market.

<sup>20</sup> See Case (1994) and Case and Mayer (1995) for early investigations into this issue for the Los Angeles and Boston markets.

While one cannot know for sure whether there was a bubble during this time period (in at least some markets), we can determine whether the data are consistent with the key implications of the theory presented above. One of the key predictions of our models is that markets with highly elastic supply sides are much less likely to have ‘bubbles’. We will examine this implication first by comparing the price gains over the 1982-1989 period in elastic and inelastic cities. Dividing the sample in two at the median value for the share of developable land based on geographic constraints (which is 88%) finds a large difference in real price appreciation between the less versus more elastic areas. On average, the relatively inelastic markets appreciated nearly five times more in real terms (23.2% versus 5.0%) than the relatively elastic markets over this period.

Regressing each metropolitan area’s price appreciation on our supply side proxy finds a statistically and economically significant relationship in which a more elastic supply side is associated with materially lower real price growth during this boom. These results are presented in column one of Table 3. The coefficient value of -0.67 (std. err. = 0.17) implies that a one standard deviation change in the share of developable land (which equals 20 percentage points and corresponds to a move from the 25<sup>th</sup> to 50<sup>th</sup> or the 50<sup>th</sup> to 75<sup>th</sup> percentiles of the distribution for this variable) is associated with about a 13 percentage point reduction in real appreciation during this time period. This finding is consistent with the model’s implication that more inelastic places will have bigger price changes during bubbles, but it is also compatible with the view that the rise in prices in the 1980s was driven by fundamentals. Inelasticity should produce bigger price increases, whether the inelasticity is driven by rational or irrational forces.

Perhaps the biggest challenge to the implication of this simple bivariate regression is that it reflects demand-side rather than supply-side differences across markets. To investigate this further, we include different measures of housing demand as controls. Two amenity variables—warmth (mean January temperature) and dryness (mean annual precipitation)—were used. While these particular variables don’t change over time, demand for them might have, so they provide a natural way of controlling for changes in demand. We also explored a variety of economic variables including the share of the population with college degrees in 1980, the level of real income in 1980 as measured by the Bureau of Economic Analysis, and the growth in that income (between 1982 and 1989 for this particular regression). The initial level of educational achievement and 1980 income are static, of course, and should only matter if they are correlated with changes in local economies over the 1980s. Change in BEA real income gives us a measure of income growth.

Column 2 of Table 3 reports coefficient estimates on our supply side proxy based on a specification that includes these demand controls. The coefficient falls to -0.52 (std. err. = 0.17), but remains statistically significant.<sup>21</sup> There was much heterogeneity in house price growth across markets during the 1980s boom. Supply side conditions certainly do not explain all that cross sectional variation, but they can account for a significant fraction. Essentially, a one standard deviation change in the degree of supply-side constraint is associated with a 0.3 standard deviation change in real price appreciation during the boom.

The last two regressions reported in Table 3 examine quantity, using the logarithm of the number of permits between 1982 and 1989 as the dependent variable. In this case, we also control for the logarithm of the number of homes in the area in 1980 and the logarithm of the

---

<sup>21</sup> It is the economic variables, both 1980 income and its growth from 1982-1989, which are the most highly significant and account for the attenuation of the coefficient on the developable land share. All results are available upon request.

land area of the metropolitan area. The logarithmic formulation means that we can interpret these results as representing permits relative to the initial stock of housing or permits relative to land area. In general, the relationship between the increase in the number of housing units and the supply side proxy is less robust. The baseline regression finds a statistically insignificant coefficient of the wrong sign (column 3). Adding in controls in the next specification, changes the coefficient to 0.007 (std. err. = 0.003), which means that a one standard deviation higher fraction of developable land is associated with a 0.14 log point increase in our permitting variable, which amounts to just over 0.16 standard deviations for that variable. Thus, there is modest evidence consistent with Proposition 3, which suggests that more elastic places build more housing during the boom.

These results indicate that the more inelastic markets experienced a much larger price response and a somewhat smaller quantity response during the boom period of the 1980s. The distributional data reported in Table 4 yield more insight into the full extent of the ability of elasticity to explain price growth variation during this time period. We split the sample into three groups based on our developable land proxy for elasticity.<sup>22</sup>

The mean differences across the groups are very large, as expected. Being relatively inelastic certainly is no guarantee of high price growth. One always needs strong demand for that, and not all areas with relatively low shares of developable land have growing demand. However, being relatively elastic really does seem to have limited the ability of a market to experience a truly large price boom during this period. The data in Table 4 show that almost none of the most elastic metropolitan areas experienced real price appreciation of at least 33%,

---

<sup>22</sup> The most elastic third of markets is slightly larger because of lumpiness at the cut-off margin for the share of developable land. Nothing is affected if we move those marginal metros into the middle group.



which is the lower bound for the top quartile of markets. During this decade, really high price growth over a long period of time required some type of constraint on development.

### *The 1989-1996 Bust*

The case for interpreting 1982-1989 as a bubble in those inelastic places is made stronger by the enormous mean reversion over the next seven years. Figure 4 plots the price changes between 1990 and 1996 on the price growth between 1982 and 1989.<sup>23</sup> Overall, the correlation is -59 percent. Whatever was pushing prices up in the 1980s seems to have disappeared in the 1990s, and it is challenging to identify real economic factors that can explain that change.

Mean differences in price appreciation between relatively inelastic and elastic areas were much smaller during the housing bust. The forty markets with developable land shares below the sample median did suffer larger real price declines on average, but the gap is not particularly large: -7.9% for the relatively constrained markets versus -2.7% for the unconstrained group. There are more extreme declines among the inelastic set. One quarter of them suffered real price declines of at least -26%. The analogous figure for the elastic markets is only -9%.

The regression results are not nearly as robust as those for the boom period. These are reported in Table 5, which replicates Table 3 for the 1989-1996 time frame and then adds two specifications. The baseline bivariate regression reported in column one yields a small, positive coefficient. However, including the demand controls reduces its magnitude and eliminates all statistical significance (column 2). In the third specification, we include a control for price growth during the 1982-1989 boom period and the interaction between the

---

<sup>23</sup> We use non-overlapping years to minimize the mean reversion that would be created by pure measurement error in 1989 prices.

degree of price growth during the boom and the amount of available land. This makes the supply elasticity proxy highly statistically significant ( $t=-5.8$ ), so that on average, the extent of real price changes falls with supply elasticity. The coefficient on the interaction term is small enough so that the marginal effect of an added percentage point of developable land is negative throughout most of the range of the data.<sup>24</sup>

While there is substantial mean reversion on average, the positive coefficient on the interaction term indicates that there is less mean reversion in more elastic markets. The coefficient magnitudes imply that there is mean reversion even in markets with 100% developable land shares, but this still is consistent with the implications of the model above that the post-bubble drops in house prices should be greater in the more inelastic markets.

The first two permit regressions for this time period yield results similar to those reported in Table 3. There is modest evidence in favor of more elastic markets experiencing a larger quantity response during the bust. The last permit regression again controls for price growth during the previous boom and its interaction with the share of developable land. On average, as the model predicts, greater real price appreciation in a metropolitan area during the boom is associated with less permitting intensity during the bust. However, the positive coefficient on the interaction term indicates that this negative impact is weakened the greater is the elasticity of supply.<sup>25</sup>

While the average effect of the supply side is very small (and negative), the combined effect with the interaction term finds that the derivative of permitting intensity during the bust with respect to the share of developable land is positive as long as real appreciation during the

---

<sup>24</sup> The relevant derivative turns positive only if price growth during the previous boom was above 58.6%. That is near the 90<sup>th</sup> percentile of the distribution for that variable.

<sup>25</sup> The marginal effect actually turns positive for markets with developable land shares above 88%, which is near the sample median for that variable. Hence, in the more elastically supplied markets, a small increase in the previous boom is associated with more, not less, permitting.

boom exceeded 5.4%. This is well below sample average price growth of 14.6% (median=8.2%, with a 25<sup>th</sup> percentile value of -5.4%) for that period, so this specification indicates that the quantity response was higher the more elastic the market, as long as the market has enough underlying demand growth that its prices actually rose in real terms during the boom period.

## **V. The Post-1996 Boom**

In some ways, the years after 1996 mirrored the boom of the 1980s. However, the post-1996 boom was bigger and more widespread. Every one of our 79 markets experienced real price gains between 1996 and 2006. Mean real price growth over this period was 57.9%. However, the price gains continued to be much higher in more inelastic places on average. The average price appreciation difference between areas above or below the sample median amount of developable land is a whopping 47 percentage points. Real price appreciation averaged 81% in the relatively inelastic markets and 34% in the relatively elastic markets. Table 6's breakdown of the distribution of real price growth for each third of our metropolitan areas based on their share of developable land further confirms this picture.

Table 7's regression results for this period show a very strong negative relation between price appreciation and our supply side proxy. The coefficient from the second specification that includes all our demand controls is one quarter larger than that reported in Table 3 for the 1980s boom. Supply elasticity correlates strongly with the extent of price growth in booms. The findings in the final two columns of this table again show there is more construction in elastic places in booms. The magnitudes also are similar to those reported in Table 3 from the 1980s boom.

However, there is one striking way in which the post-1996 boom differed from the boom of the 1980s: there was exceptional price growth in a modest number of relatively or highly elastic places. Of the 19 metropolitan areas that experienced at least a doubling of real prices (100%+) over this period, one (Orlando-Kissimmee) is in the most elastic group and five are in the middle group (Fresno, Phoenix-Mesa-Scottsdale, Stockton, Tampa-St. Petersburg-Clearwater, and Washington-Arlington-Alexandria).

One possible explanation of this fact is that supply had become much less elastic in some of the areas that we classified as elastic based on Saiz's (2008) data. To check this, we look at permitting intensity. Figure 5 plots the sample average across our 79 metropolitan areas of annual permits as a share of their 1990 housing stocks. Permitting intensity clearly cycles, with permits as a fraction of the stock in the middle of our sample period being as low as 1.1% in the depth of a bust and as high as 2.6% in 2005.

Figures 6-11 then plot the analogous data for the six aforementioned markets that experienced huge real price growth in the recent boom, but are not constrained by our supply side measure. In each market other than Washington-Arlington-Alexandria, permitting intensity was as high or higher in the recent boom than in the 1980s boom. In the Phoenix and Orlando areas, annual permitting reached as high as 6% of the 1990 stock. And, permitting intensity did not fall until late in the recent boom, even for the Washington market. While permitting intensity was not high by national standards in the Washington market, it was in the others. Hence, it is difficult to make a case that anyone reasonably thought these markets were becoming inelastic in nature.

Figure 12 then plots the mean real house price-to-MPPC ratio for the Orlando metropolitan area. For two decades prior to 2003 when this ratio hit 1.22, prices in this market

hovered within 10-15% of our estimate of minimum profitable production costs. It then proceeded to grow by another 39% before peaking at 1.70 in 2006. As Figure 7 above shows, Orlando's permitting intensity had fallen between 1999-2001, but was increasing sharply between 2001 and 2003 and would reach its all-time high in 2005. Thus, land in this and a few other similar markets looks to be becoming increasingly expensive at the same time that new supply is accelerating. This well may be a more profound signal of a bubble than any other of which we are aware.

Proposition 5 above predicts that booms, to the extent they exist at all in elastic markets, will be shorter than in inelastic markets. To see whether the data are consistent with this implication of the model, we measured the length of a market's boom in the 1996-2006 period as follows. The start of the boom is defined by the first year subsequent to 1996 in which the area's price-to-MPPC ratio had increased by at least 20% over its 1996 value. The 20% number was chosen because we believe it is large enough to ensure that house prices really have risen substantially relative to fundamental production costs in a way that cannot be accounted for by measurement error. The length of the boom is then determined by the number of years before the price-to-MPPC ratio begins to turn down.

Among the most elastic third of markets with the highest shares of developable land, eleven saw their price-to-MPPC ratio rise at least 20% above its 1996 level for at least one year. The mean number of years before that ratio began to decline was 1.7 for this group. By this definition, booms last for less than two years in the most elastic markets. Naturally, a greater share of the most inelastic third of markets experienced a boom according to this metric. Among the 20 that did, the mean number of years before a downturn in this ratio was

4.1. Thus, booms last much longer in the more inelastic markets, as predicted by Proposition 5.<sup>26</sup>

## **VI. Conclusion**

Housing price volatility can be quite high, and in some cases it appears to be too high to be explained by changes in the fundamentals of the housing market. This seems to suggest that financial market wisdom regarding bubbles and overoptimism can be imported into housing economics. However, it is critical to incorporate the supply side when analyzing housing markets. As a factual matter, the supply of housing is not fixed, and the possibility of oversupply of housing during a boom is one of the primary ways in which housing bubbles may create substantial welfare losses.

In this paper, we presented a simple model of housing bubbles with endogenous housing supply. On some matters, the model gave clear predictions. Housing prices should increase more during bubbles in places where housing supply is inelastic. Bubbles should be more common and longer in places where supply is inelastic. On other issues, the implications of the model were less clear, including with respect to the impact of bubbles on the amount of new construction and social welfare.

We then examined price and quantity increases during the two large national price swings of the past 25 years. During both price booms, price increases were higher in places where housing supply was more inelastic because of steep topography, just as the model suggests. Moreover, most elastic places seem to have avoided having a bubble entirely during the 1980s. Over the last decade, there were more modestly elastic places that also experienced

---

<sup>26</sup> The mean length of boom for the 15 markets from the middle third of the developable land share distribution that experienced at least a 20% rise in their price-to-MPPC ratio is 3.3 years, in between the most and least elastic markets.

big price increases, but as the model suggests, the housing price booms in elastic places were much shorter in duration than those in inelastic places.

The empirical results on quantities built during booms were more ambiguous, just like the model's predictions. During the 1980s, once we controlled for other area-level characteristics, more elastic places built more. The same thing is even more true over the past ten years.

Our results on elasticity were weakest during the bust period. While the level of mean reversion during the bust is enormous, there is little correlation between price declines during the bust and the degree of elasticity. Places with bigger price growth during the 1980s had less construction in the early 1990s, but this effect was muted in more elastic areas. More generally, it is hard to really predict what will happen to elastic areas that experienced price booms over the last five years. In the past, places that elastic did not experience price booms. However, a reasonable expectation might be that prices in these elastic areas are likely to continue their fall downward, returning to minimum profitable production cost levels, which have been the historical norm in these areas.

## References

- Barberis, N., Ming H., Santos, T., 2001. Prospect Theory and Asset Prices. *Quarterly Journal of Economics* 116, 1-53.
- Case, K., 1994. A Decade of Boom and Bust in the Prices of Single-Family Homes: Boston and Los Angeles: 1983-1993. *New England Economic Review*, March 1994.
- Case, K., Mayer, C., 1995. The Housing Cycle in the Boston Metropolitan Area: The Boom, the Bust, and the Recovery. *New England Economic Review*, March/April 1995.
- Case, K., Shiller, R., 1989. The Efficiency of the Market for Single-Family Homes. *American Economic Review* 79 (1), 125-137.
- Delong, J. B., Shleifer, A., Summers, L., Waldmann, R., 1990. Noise Trader Risk in Financial Markets. *Journal of Political Economy* 98 (4), 703-738.
- Duranton, G., Combes, P-P., Gobillon, L., Roux, S., 2008. Estimating Agglomeration Economies with History, Geology and Worker Effects. Working paper, University of Toronto, January 30, 2008.
- Flood, R., Hodrick, R., 1990. On Testing for Speculative Bubbles. *Journal of Economic Perspectives* 4 (2), 85-101.
- Glaeser, E., Gyourko, J., 2003. The Impact of Zoning on Housing Affordability. *Economic Policy Review* 9 (2), 21-39.
- Glaeser, E., Gyourko, J., 2005. Urban Decline and Durable Housing. *Journal of Political Economy* 113 (2), 345-375.
- Glaeser, E., Gyourko, J., 2006. Housing Dynamics. NBER Working Paper 12787, revised February 2008.
- Glaeser, E., Gyourko, J., 2008. Arbitrage in Housing, in: Glaeser, E., Quigley, J. (Eds.), *Housing and the Built Environment: Access, Finance, Policy*. Lincoln Land Institute of Land Policy, Cambridge, MA, forthcoming.
- Glaeser, E., Gyourko, J., Saks, R., 2005. Why Have Housing Prices Gone Up? *American Economic Review* 95 (2) 2005: 329-333.
- Gyourko, J., Saiz, A., Summers, A., 2008. A New Measure of the Local Regulatory Environment for Housing Markets. *Urban Studies* 45 (3), 693-729.
- Hansen, L., Scheinkman, J., 1995. Back to the Future: Generating Moment Implications for Continuous-Time Markov Processes. *Econometrica* 63 (4), 767-804.



- Himmelberg, C., Mayer, C., Sinai, T., 2005. Assessing High House Prices: Bubbles, Fundamentals, and Misperceptions. *Journal of Economic Perspectives* 19 (4), 67-92.
- Holmes, T., Lee, S., 2008. Cities as Six-by-Six-Mile Squares: Zipf's Law. Mimeo. University of Minnesota, 2008.
- Hong, H., Scheinkman, J., Xiong, W., 2008. Advisors and Asset Prices: A Model of the Origin of Bubbles. *Journal of Financial Economics*, forthcoming.
- Hong, H., Scheinkman, J., Xiong, W., 2006. Asset Float and Speculative Bubbles. *Journal of Finance* 61 (3), 1073-1117.
- Hoxby, C., 2000. Does Competition among Public Schools Benefit Students and Taxpayers?, *American Economic Review* 90 (5), 1209-38.
- Linneman, P., 1986. An Empirical Test of the Efficiency of the Housing Market. *Journal of Urban Economics* 20, 140-154.
- Mayer, C., Sinai, T., 2007. Housing and Behavioral Finance, mimeo, September 25, 2007.
- McMillan, D., McDonald, J., 1991. Urban Land Value Functions with Endogenous Zoning. *Journal of Urban Economics* 29, 14-27.
- Meese, R., Wallace, N., 1994. Testing the Present Value Relation for House Prices: Should I Leave My House in San Francisco? *Journal of Urban Economics* 35, 245-266.
- Ofek, E., Richardson, M., 2003. Dotcom mania: The rise and fall of internet stock prices. *Journal of Finance* 58, 1113-1137.
- R.S. Means Company, 2008. Assemblies Cost Data 2008. R.S. Means Company, Kingston, MA.
- Rosenthal, S., 1989. Residential Building and the Cost of Construction: New Evidence on the Efficiency of the Housing Market. *Review of Economics and Statistics* 81, 288-302.
- Rosenthal, S., Strange, W., 2008. The Attenuation of Human Capital Externalities, *Journal of Urban Economics*, forthcoming.
- Saiz, A., 2008. On Local Housing Supply Elasticity, mimeo, The Wharton School, University of Pennsylvania.
- Saks, R., 2008. Job Creation and Housing Construction: Constraints on Metropolitan Area Employment Growth. *Journal of Urban Economics* 64, 178-195.
- Shiller, R., 1981. Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review* 71 (3), 421-436.

Shiller, R., 2005. *Irrational Exuberance*, second ed. Princeton University Press, Princeton, NJ.

Shleifer, A., 1985. Do Demand Curves for Stocks Slope Down? *Journal of Finance* 41 (3), 579-590.

Topel, R., Rosen, S., 1988. Housing Investment in the United States. *Journal of Political Economy* 96 (4), 718-740.

Wallace, N., 1988. The Market Effects of Zoning Undeveloped Land: Does Zoning Follow the Market? *Journal of Urban Economics* 23, 307-326.

**Table 1: House Prices and Production Costs, 2007**

Metropolitan Area	State	House Price	Production Costs
Akron	OH	\$130,611	\$165,975
Albuquerque	NM	\$208,903	\$153,136
Allentown-Bethlehem-Easton	PA	\$215,976	\$176,832
Atlanta-Sandy Springs-Marietta	GA	\$187,799	\$154,493
Austin-Round Rock	TX	\$180,222	\$137,373
Bakersfield	CA	\$219,291	\$181,320
Baltimore-Towson	MD	\$275,101	\$159,190
Baton Rouge	LA	\$143,864	\$145,516
Birmingham-Hoover	AL	\$136,861	\$153,345
Boston-Quincy	MA	\$322,721	\$200,214
Bridgeport-Stamford-Norwalk	CT	\$498,022	\$187,479
Buffalo-Niagara Falls	NY	\$122,500	\$175,997
Charlotte-Gastonia-Concord	NC	\$173,802	\$138,626
Chicago-Naperville-Joliet	IL	\$259,812	\$194,682
Cincinnati-Middletown	OH	\$143,263	\$159,086
Cleveland-Elyria-Mentor	OH	\$139,847	\$172,134
Colorado Springs	CO	\$203,942	\$159,295
Columbia	SC	\$137,075	\$137,478
Columbus	OH	\$149,531	\$161,904
Dallas-Plano-Irving	TX	\$136,368	\$144,680
Dayton	OH	\$120,640	\$155,954
Denver-Aurora	CO	\$217,760	\$162,739
Detroit-Livonia-Dearborn	MI	\$107,097	\$180,485
Fort Lauderdale-Pompano Beach-Deerfield Beach	FL	\$303,956	\$151,988
Fort Wayne	IN	\$103,013	\$154,180
Fort Worth-Arlington	TX	\$115,798	\$140,714
Fresno	CA	\$244,810	\$184,452
Gary	IN	\$140,650	\$172,656
Grand Rapids-Wyoming	MI	\$131,518	\$143,219
Greensboro-High Point	NC	\$134,276	\$137,478
Harrisburg-Carlisle	PA	\$166,348	\$166,393
Houston-Sugar Land-Baytown	TX	\$125,962	\$152,405
Indianapolis-Carmel	IN	\$135,988	\$160,860
Jacksonville	FL	\$199,607	\$145,098
Kansas City	MO	\$135,415	\$176,414
Las Vegas-Paradise	NV	\$283,653	\$174,013
Little Rock-North Little Rock-Conway	AR	\$120,840	\$147,916
Los Angeles-Long Beach-Glendale	CA	\$514,650	\$185,287
Louisville-Jefferson County	KY	\$133,783	\$157,520
Memphis	TN	\$114,848	\$149,378
Miami-Miami Beach-Kendall	FL	\$330,573	\$155,432
Milwaukee-Waukesha-West Allis	WI	\$190,099	\$176,310
Minneapolis-St. Paul-Bloomington	MN	\$216,199	\$192,594
Nashville-Davidson--Murfreesboro--Franklin	TN	\$176,949	\$150,735
Newark-Union	NJ	\$395,231	\$193,325
New Orleans-Metairie-Kenner	LA	\$159,207	\$151,570
New York-White Plains-Wayne	NY	\$720,863	\$224,641

**Table 1: House Prices and Production Costs, 2007 (continued)**

Metropolitan Area	State	House Price	Production Costs
Oakland-Fremont-Hayward	CA	\$514,768	\$209,922
Oklahoma City	OK	\$110,531	\$141,549
Omaha-Council Bluffs	NE	\$125,899	\$156,894
Orlando-Kissimmee	FL	\$230,099	\$154,597
Oxnard-Thousand Oaks-Ventura	CA	\$513,529	\$182,886
Philadelphia	PA	\$219,469	\$196,665
Phoenix-Mesa-Scottsdale	AZ	\$258,467	\$154,180
Pittsburgh	PA	\$116,960	\$170,360
Providence-New Bedford-Fall River	RI	\$264,245	\$182,051
Raleigh-Cary	NC	\$203,291	\$137,895
Richmond	VA	\$198,628	\$153,762
Riverside-San Bernardino-Ontario	CA	\$332,515	\$182,364
Rochester	NY	\$122,900	\$170,673
St. Louis	MO	\$143,323	\$178,084
Salt Lake City	UT	\$273,874	\$150,944
San Antonio	TX	\$118,524	\$142,384
San Diego-Carlsbad-San Marcos	CA	\$447,962	\$181,216
San Francisco-San Mateo-Redwood City	CA	\$721,694	\$209,922
San Jose-Sunnyvale-Santa Clara	CA	\$678,274	\$209,922
Seattle-Bellevue-Everett	WA	\$417,512	\$178,919
Stockton	CA	\$264,884	\$186,018
Tacoma	WA	\$289,587	\$176,101
Tampa-St. Petersburg-Clearwater	FL	\$192,969	\$158,668
Toledo	OH	\$116,206	\$168,689
Tucson	AZ	\$222,790	\$149,378
Tulsa	OK	\$107,404	\$138,313
Vallejo-Fairfield	CA	\$342,161	\$194,055
Warren-Troy-Farmington Hills	MI	\$176,499	\$174,118
Washington-Arlington-Alexandria	DC	\$403,374	\$170,255
West Palm Beach-Boca Raton-Boynton Beach	FL	\$295,560	\$155,432
Wichita	KS	\$105,162	\$144,785
Wilmington	DE	\$245,042	\$181,007

<b>Table 2: Summary Statistics on Percentage Price Changes Over Time 79 Metropolitan Area Sample</b>						
	1982-1989		1989-1996		1996-2006	
	Nominal	Real(\$07)	Nominal	Real(\$07)	Nominal	Real(*\$07)
Mean	47.2%	14.6%	19.8%	-5.3%	100.0%	57.9%
Std. Dev.	42.0%	32.6%	19.3%	16.5%	60.2%	47.9%
10 <sup>th</sup>	-1.7%	-23.5%	-6.8%	-28.3%	40.7%	11.0%
25 <sup>th</sup>	21.5%	-5.4%	4.6%	-17.8%	49.8%	18.0%
50 <sup>th</sup>	39.1%	8.2%	21.7%	-3.7%	69.5%	33.9%
75 <sup>th</sup>	70.9%	33.0%	32.5%	5.8%	146.1%	94.3%
90 <sup>th</sup>	114.8%	67.1%	40.7%	12.8%	203.3%	140.1%

**Table 3: Real Price Appreciation, Building, and Supply Conditions: 1980's Boom**

	Real Price Appreciation 1982-1989	Real Price Appreciation 1982-1989	Log of Total Permits 1982-89	Log of Total Permits 1982-89
Share of Buildable Land	-0.67 (0.17)	-0.52 (0.17)	-0.005 (0.003)	0.007 (0.003)
Other Covariates		Percent College Grads January Temperature Personal Income 1980 Income Growth 1982-89 Precipitation	Log Land Area Log 1980 Stock	Percent College Grads January Temperature Personal Income 1980 Income Growth 1982-89 Precipitation Log Land Area Log 1980 Stock
N	79	79	79	79
R-Squared	0.17	0.58	0.58	0.79

**Table 4: Distribution of Real Price Growth in the 1980s Boom  
by Degree of Supply Constraint**

	n=26	n=25	n=28
	Most Inelastic	Middle Third	Most Elastic
Mean	28.6%	12.5%	3.4%
Std. Dev.	36.8%	28.2%	28.2%
10 <sup>th</sup>	-9.0%	-24.0%	-30.5%
25 <sup>th</sup>	-2.3%	-5.4%	-18.3%
50 <sup>th</sup>	23.2%	8.7%	5.0%
75 <sup>th</sup>	64.1%	33.8%	12.5%
90 <sup>th</sup>	86.1%	50.8%	33.0%

**Table 5: Real Price Appreciation, Building, and Supply Conditions: 1990's Bust**

	Real Price Appreciation 1989-1996	Real Price Appreciation 1989-1996	Real Price Appreciation 1989-1996	Log of Total Permits 1990-96	Log of Total Permits 1990-96	Log of Total Permits 1990-96
Share of Buildable Land	0.24 (0.09)	-0.05 (0.09)	-0.41 (0.07)	0.0037 (0.0028)	0.0065 (0.0037)	-0.0020 (0.0036)
Real Price Growth 1982-1989			-0.86 (0.12)			-0.0328 (0.0063)
Interaction of 1982-89 Price Growth and Buildable Land Share			0.007 (0.001)			0.00037 (0.00008)
Other Covariates		Percent College Grads January Temperature Personal Income 1980 Income Growth 1989-96 Precipitation	Percent College Grads January Temperature Personal Income 1980 Income Growth 1989-96 Precipitation	Log Land Area Log 1990 Stock	Percent College Grads January Temperature Personal Income 1980 Income Growth 1989-96 Precipitation Log Land Area Log 1990 Stock	Percent College Grads January Temperature Personal Income 1980 Income Growth 1989-96 Precipitation Log Land Area Log 1990 Stock
N	79	79	79	78	78	78
R-Squared	0.09	0.47	0.82	0.65	0.68	0.77

**Table 6: Distribution of Real Price Growth in the 1996-2006 Boom  
by Degree of Supply Constraint**

	n=26	n=25	n=28
	Most Inelastic	Middle Third	Most Elastic
Mean	93.9%	53.8%	28.2%
Std. Dev.	50.4%	39.7%	26.3%
10 <sup>th</sup>	13.5%	15.3%	6.2%
25 <sup>th</sup>	49.4%	24.7%	12.6%
50 <sup>th</sup>	98.6%	36.2%	18.4%
75 <sup>th</sup>	140.2%	86.6%	30.1%
90 <sup>th</sup>	146.5%	115.1%	67.0%

**Table 7: Real Price Appreciation, Building, and Supply Conditions: 1996-2006 Boom**

	Real Price Appreciation 1996-2006	Real Price Appreciation 1996-2006	Log of Total Permits 1997-2006	Log of Total Permits 1997-2006
Share of Buildable Land	-1.54 (0.21)	-0.77 (0.23)	0.0040 (0.0030)	0.0071 (0.0035)
Other Covariates		Percent College Grads January Temperature Personal Income 1980 Income Growth 1996-06 Precipitation	Log Land Area Log 1990 Stock	Percent College Grads January Temperature Personal Income 1980 Income Growth 1996-06 Precipitation Log Land Area Log 1990 Stock
N	79	79	78	78
R-Squared	0.40	0.63	0.67	0.79



Figure 1: Price-to-Cost Ratio, 79 Metro Average

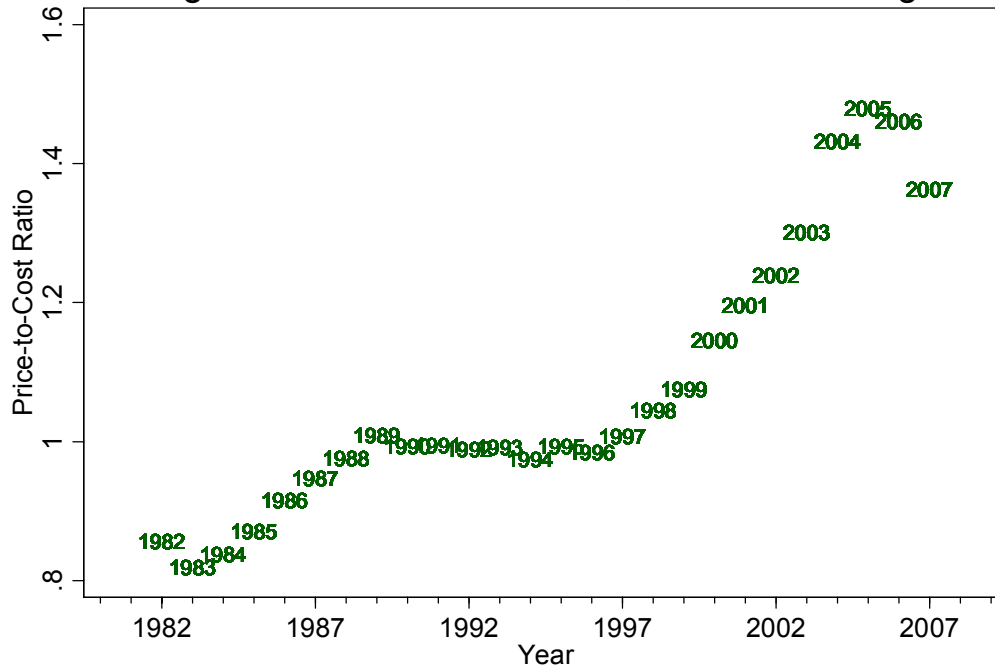


Figure 2: RealAverage Prices, 79 Metro Average

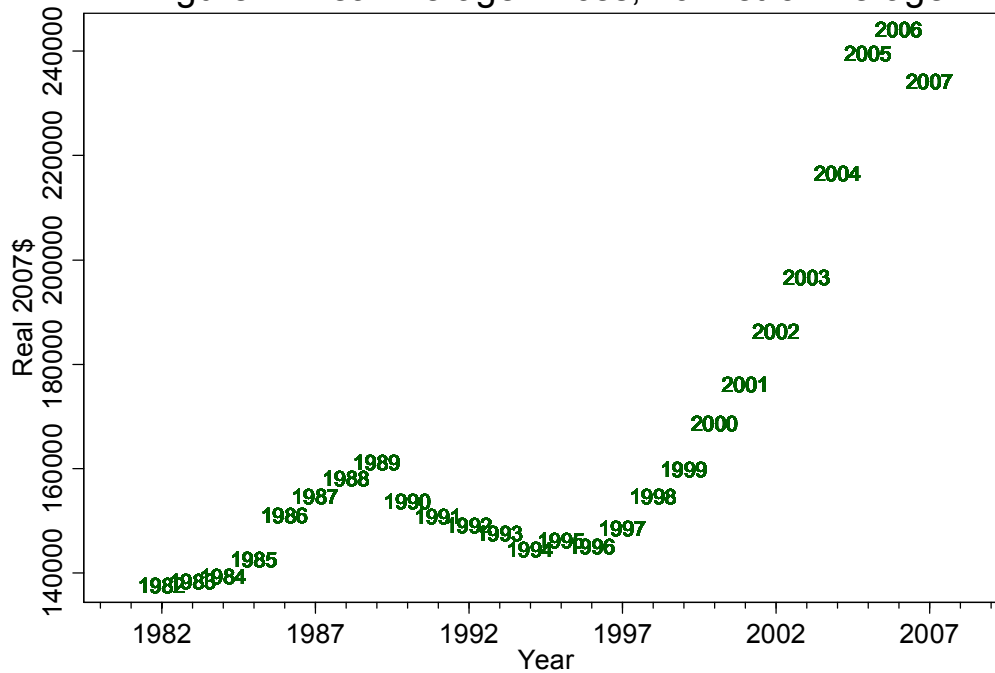


Figure 3: Aggregate Permits Over 79 Metros

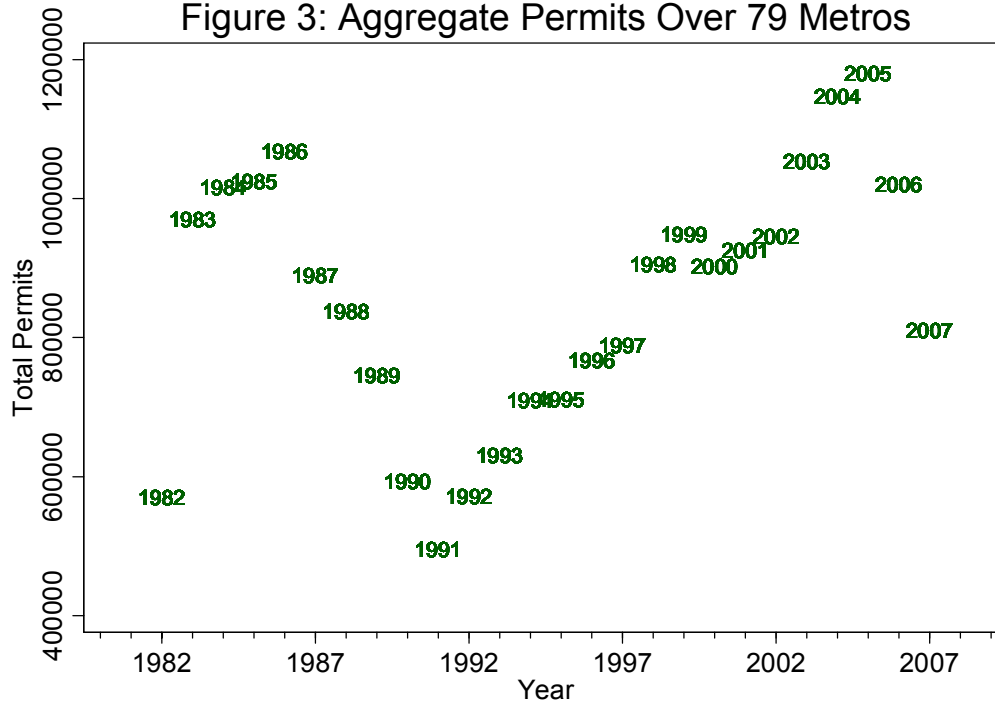


Figure 4: Mean Reversion in Prices, 79 Metro Average

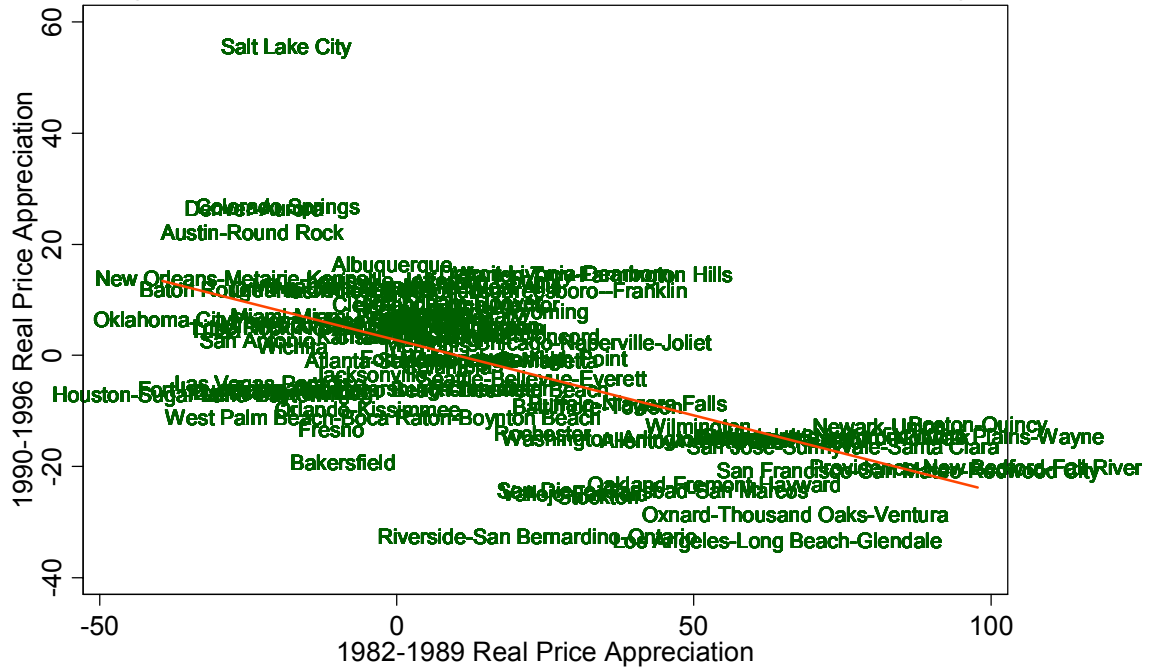


Figure 5: Average Permitting Intensity, 79 Metro Sample

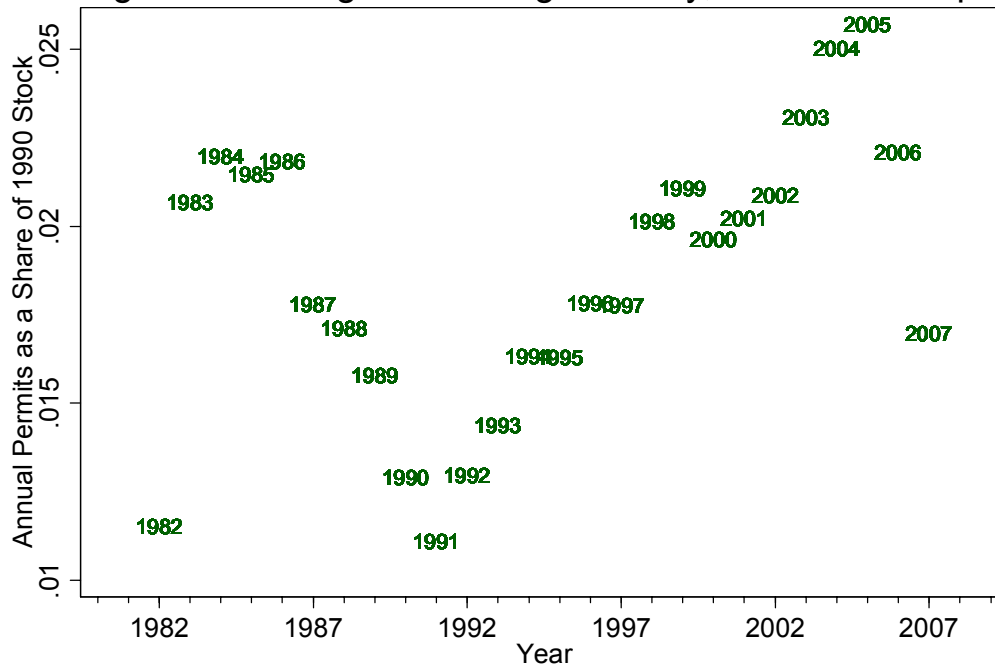


Figure 6: Average Permitting Intensity, Fresno

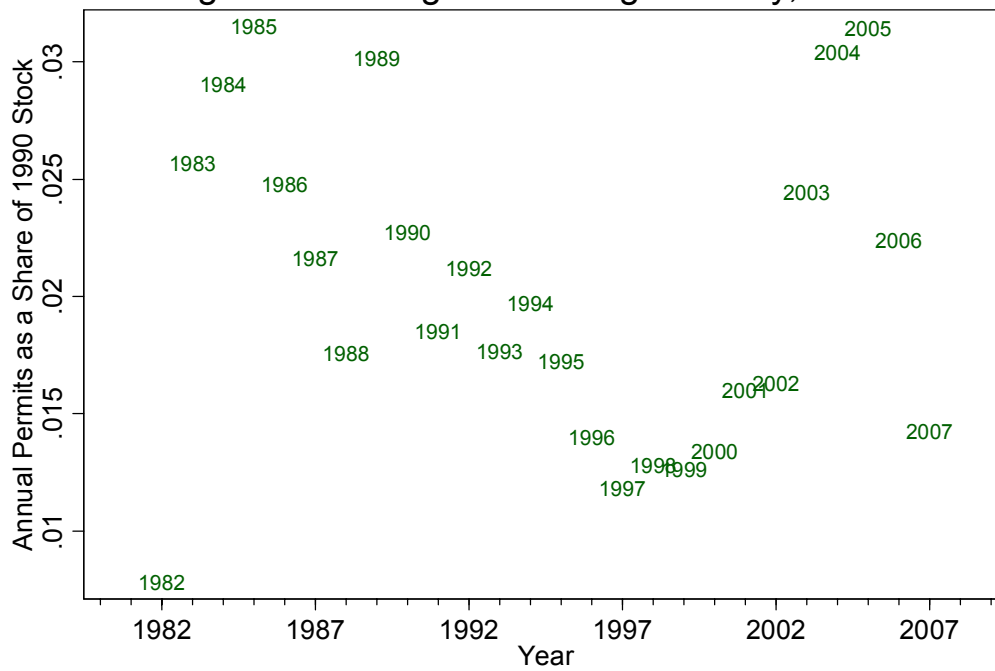


Figure 7: Average Permitting Intensity, Orlando

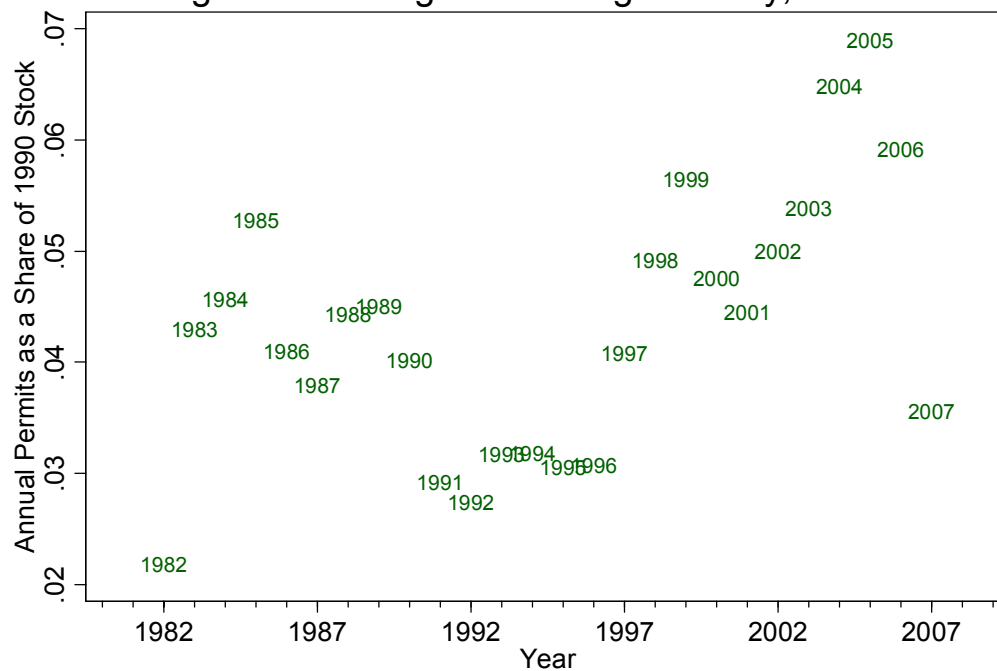


Figure 8: Average Permitting Intensity, Phoenix

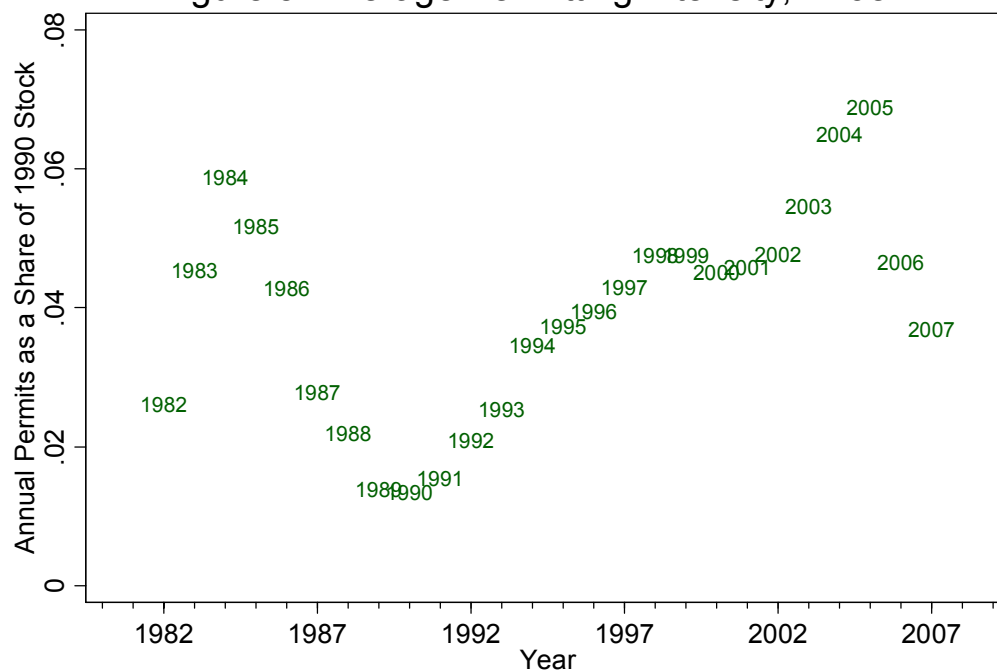


Figure 9: Average Permitting Intensity, Stockton

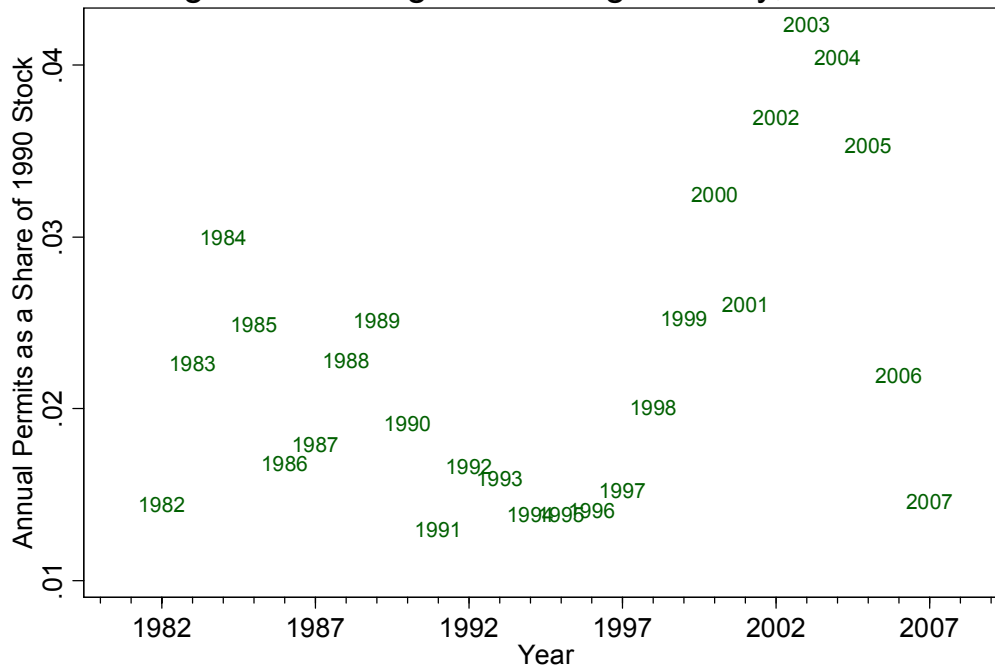


Figure 10: Average Permitting Intensity, Tampa

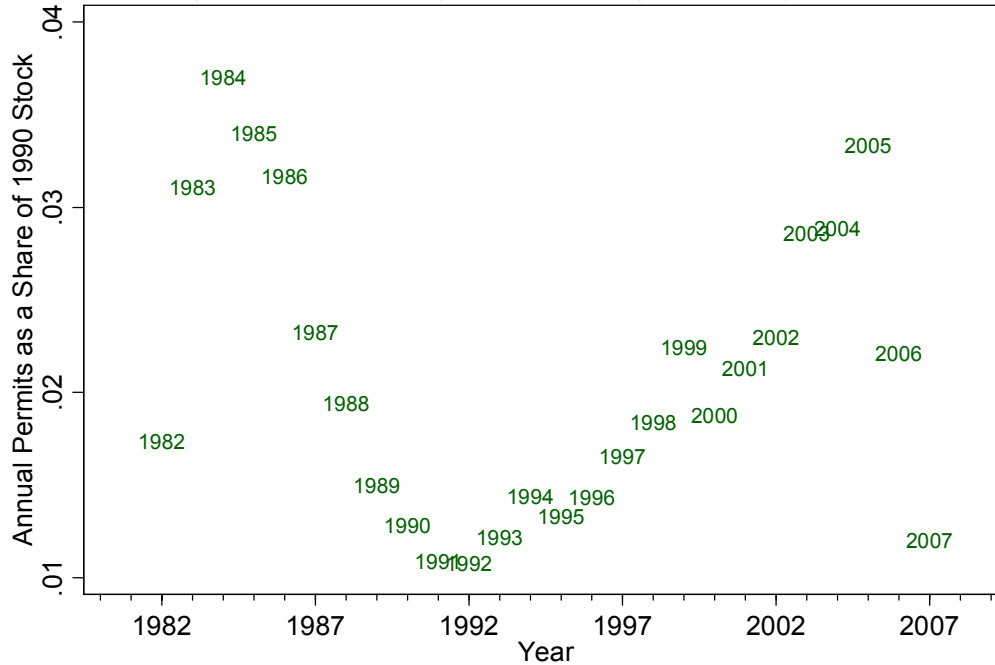


Figure 11: Average Permitting Intensity, Washington DC

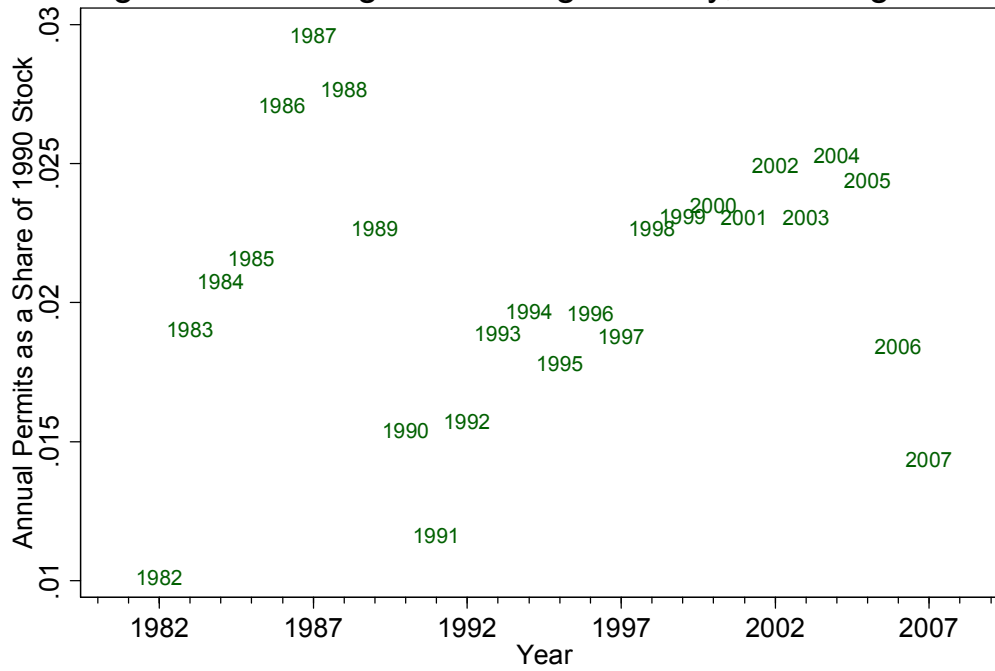
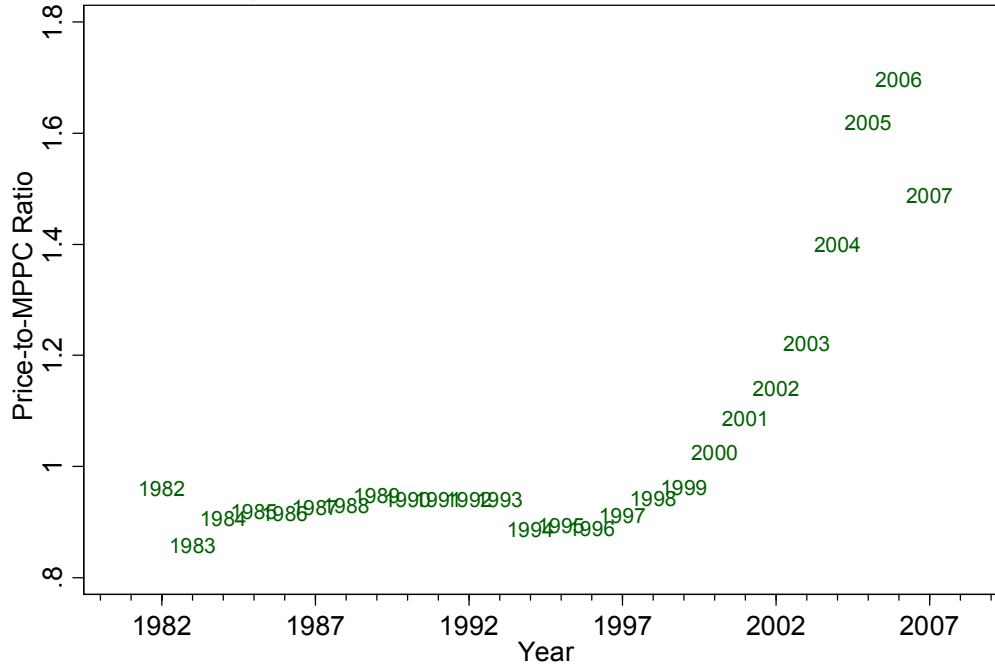


Figure 12: Orlando Price-to-MPPC Ratio



## Appendix 1: Proofs of Propositions

*Proof of Proposition 1:* If  $P(t+j) > c_0 + \varepsilon$  for all positive  $j$ , then there construction levels will be greater than  $\varepsilon / c_1$  for all positive  $j$ , which implies an infinite amount of construction and an infinite city size which means that there will be more homes for sale than buyers and the equilibrium price will be zero which contradicts the initial assumption.

*Proof of Proposition 2:* During the bubble:

$$H(t+j) = e^{-\gamma j} H(t) + (1 - e^{-\gamma j}) \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} + \frac{\lambda \bar{\theta}}{\gamma((r+\lambda)c_1 + v_1)} \right)$$

$$I(t+j) = e^{-\gamma j} \left( \gamma \frac{a_0 + v_0 - rc_0}{v_1 \lambda} + \frac{\lambda \bar{\theta}}{(r+\lambda)c_1 + v_1} - \gamma H(t) \right)$$

$$P(t+j) = c_0 + c_1 \gamma e^{-\gamma j} \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} + \frac{\lambda \bar{\theta}}{\gamma((r+\lambda)c_1 + v_1)} - H(t) \right)$$

For  $j < k$ ,  $\frac{\partial I(t+j)}{\partial \bar{\theta}} = \frac{e^{-\gamma j} \lambda}{(r+\lambda)c_1 + v_1} > 0$  and  $\frac{\partial H(t+j)}{\partial \bar{\theta}} = \frac{(1 - e^{-\gamma j}) \lambda}{\gamma((r+\lambda)c_1 + v_1)} > 0$  and

$\frac{\partial P(t+j)}{\partial \bar{\theta}} = \frac{c_1 e^{-\gamma j} \lambda}{(r+\lambda)c_1 + v_1} > 0$ , all of which are unambiguously positive. As the length of

the bubble increases, the impact of the shock on investment and prices declines, but the impact of the shock on the size of the housing stock increases. Differentiation gives us

$$\frac{\partial^2 I(t+j)}{\partial \bar{\theta} \partial j} = \frac{-\gamma e^{-\gamma j} \lambda}{(r+\lambda)c_1 + v_1} < 0 \text{ and } \frac{\partial^2 H(t+j)}{\partial \bar{\theta} \partial j} = \frac{e^{-\gamma j} \lambda}{(r+\lambda)c_1 + v_1} > 0 \text{ and}$$

$$\frac{\partial^2 P(t+j)}{\partial \bar{\theta} \partial j} = \frac{-c_1 \gamma^2 e^{-\gamma j} \lambda}{\gamma((r+\lambda)c_1 + v_1)} = \frac{-c_1 \gamma e^{-\gamma j} \lambda}{(r+\lambda)c_1 + v_1} < 0.$$

After the bubble, for all  $i \geq 0$ ,

$$I(t+k+i) = \gamma e^{-\gamma(k+i)} \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} - H(t) \right) - \frac{e^{-\gamma i} (1 - e^{-\gamma k}) \lambda \bar{\theta}}{(r+\lambda)c_1 + v_1}$$

$$H(t+k+i) = e^{-\gamma(k+i)} H(t) + (1 - e^{-\gamma(k+i)}) \left( \frac{a_0 + v_0 - rc_0}{v_1 \lambda} \right) + \frac{e^{-\gamma i} (1 - e^{-\gamma k}) \lambda \bar{\theta}}{\gamma((r+\lambda)c_1 + v_1)}$$

and

$$P(t+k+i) = c_0 + c_1 \gamma e^{-\gamma(k+i)} \left( \frac{a_0 + v_0 - r c_0}{v_1 \lambda} - H(t) \right) - c_1 \frac{e^{-\gamma i} (1 - e^{-\gamma k}) \lambda \bar{\theta}}{(r + \lambda) c_1 + v_1}$$

$$\frac{\partial I(t+k+i)}{\partial \bar{\theta}} = - \frac{e^{-\gamma i} (1 - e^{-\gamma k}) \lambda}{(r + \lambda) c_1 + v_1} < 0 \text{ and } \frac{\partial H(t+k+i)}{\partial \bar{\theta}} = \frac{e^{-\gamma i} (1 - e^{-\gamma k}) \lambda}{\gamma((r + \lambda) c_1 + v_1)} > 0 \text{ and}$$

$$\frac{\partial P(t+k+i)}{\partial \bar{\theta}} = -c_1 \frac{e^{-\gamma i} (1 - e^{-\gamma k}) \lambda}{(r + \lambda) c_1 + v_1} < 0.$$

The cross partial derivatives with respect to the duration of the bubble are

$$\frac{\partial^2 I(t+k+i)}{\partial \bar{\theta} \partial k} = - \frac{e^{-\gamma i} e^{-\gamma k} \gamma \lambda}{(r + \lambda) c_1 + v_1} < 0 \text{ and } \frac{\partial^2 H(t+k+i)}{\partial \bar{\theta} \partial k} = \frac{e^{-\gamma i} e^{-\gamma k} \lambda}{(r + \lambda) c_1 + v_1} > 0 \text{ and}$$

$$\frac{\partial^2 H(t+j)}{\partial \bar{\theta} \partial k} = -c_1 \frac{e^{-\gamma i} e^{-\gamma k} \gamma \lambda}{(r + \lambda) c_1 + v_1} < 0.$$

*Proof of Proposition 3:* For  $j \leq k$ , differentiation yields:  $\frac{\partial I(t+j)}{\partial \bar{\theta}} = \frac{e^{-\gamma j} \lambda}{(r + \lambda) c_1 + v_1} > 0$ ,

$$\frac{\partial H(t+j)}{\partial \bar{\theta}} = \frac{(1 - e^{-\gamma j}) \lambda}{\gamma((r + \lambda) c_1 + v_1)} > 0 \text{ and } \frac{\partial P(t+j)}{\partial \bar{\theta}} = \frac{c_1 \gamma e^{-\gamma j} \lambda}{\gamma((r + \lambda) c_1 + v_1)} > 0. \text{ Since}$$

$$\gamma = \sqrt{.25r^2 + \frac{\lambda(r + \lambda)v_1}{(r + \lambda)c_1 + v_1}} - .5r, \quad \frac{\partial \gamma}{\partial c_1} = \frac{-\lambda(r + \lambda)^2 v_1}{(2\gamma + r)((r + \lambda)c_1 + v_1)^2} < 0 \text{ and}$$

$$\frac{\gamma(2\gamma + r)((r + \lambda)c_1 + v_1)}{\lambda(r + \lambda)v_1} = \frac{(r + \lambda)c_1 + v_1}{\lambda(r + \lambda)v_1} \left( .5r^2 + 2 \frac{\lambda(r + \lambda)v_1}{(r + \lambda)c_1 + v_1} - r \sqrt{.25r^2 + \frac{\lambda(r + \lambda)v_1}{(r + \lambda)c_1 + v_1}} \right)$$

$$= 1 + \frac{(r + \lambda)c_1 + v_1}{\lambda(r + \lambda)v_1} \left( .5r^2 + \frac{\lambda(r + \lambda)v_1}{(r + \lambda)c_1 + v_1} - r \sqrt{.25r^2 + \frac{\lambda(r + \lambda)v_1}{(r + \lambda)c_1 + v_1}} \right) > 1$$

$$\text{, we know that } - \frac{c_1}{\gamma} \frac{\partial \gamma}{\partial c_1} = \frac{\lambda(r + \lambda)v_1}{\gamma(2\gamma + r)((r + \lambda)c_1 + v_1)} \frac{(r + \lambda)c_1}{(r + \lambda)c_1 + v_1} < \frac{(r + \lambda)c_1}{(r + \lambda)c_1 + v_1}.$$

Further differentiation then yields that:

$$\frac{\partial^2 I(t+j)}{\partial \bar{\theta} \partial c_1} = \frac{- \frac{\partial \gamma}{\partial c_1} e^{-\gamma j} j \lambda ((r + \lambda) c_1 + v_1) - (r + \lambda) e^{-\gamma j} \lambda}{((r + \lambda) c_1 + v_1)^2}$$

This is negative if and only if  $\frac{(r + \lambda) c_1 + v_1}{\lambda(r + \lambda) v_1} \sqrt{.25r^2 + \frac{\lambda(r + \lambda) v_1}{(r + \lambda) c_1 + v_1}} > j$ .

The cross partial effect on the housing supply equals



$$\frac{\partial H(t+j)}{\partial \bar{\theta} \partial c_1} = \lambda \frac{-\frac{c_1}{\gamma} \frac{\partial \gamma}{\partial c_1} (1 - e^{-j} - j\gamma e^{-j}) - (1 - e^{-j}) \frac{(r+\lambda)c_1}{(r+\lambda)c_1 + v_1}}{c_1 \gamma ((r+\lambda)c_1 + v_1)}$$

which is always negative because  $\frac{(r+\lambda)c_1}{(r+\lambda)c_1 + v_1} > -\frac{c_1}{\gamma} \frac{\partial \gamma}{\partial c_1}$

$$\text{The impact on prices is } \frac{\partial^2 P(t+j)}{\partial \bar{\theta} \partial c_1} = \lambda \frac{v_1 e^{-j} - c_1 e^{-j} j ((r+\lambda)c_1 + v_1) \frac{\partial \gamma}{\partial c_1}}{((r+\lambda)c_1 + v_1)^2} > 0.$$

After the bubble at time k,

$$\frac{\partial^2 I(t+k)}{\partial \bar{\theta} \partial c_1} = \frac{e^{-jk} k \left( -\frac{\partial \gamma}{\partial c_1} \right) \lambda ((r+\lambda)c_1 + v_1) + (1 - e^{-jk}) \lambda (r+\lambda)}{((r+\lambda)c_1 + v_1)^2} > 0, \text{ and}$$

$$\frac{\partial^2 P(t+k)}{\partial \bar{\theta} \partial c_1} = \lambda \frac{ke^{-jk} c_1 ((r+\lambda)c_1 + v_1) \left( -\frac{\partial \gamma}{\partial c_1} \right) - v_1 (1 - e^{-jk})}{((r+\lambda)c_1 + v_1)^2}$$

which is negative if and only if  $\frac{e^{jk} - 1}{k} > \frac{\lambda(r+\lambda)^2}{\left( r + \lambda + \frac{v_1}{c_1} \right) \sqrt{.25r^2 + \frac{\lambda(r+\lambda)v_1}{(r+\lambda)c_1 + v_1}}}$ . As  $c_1$

goes to zero, the right hand side of the expression also goes to zero so the condition must hold. As  $c_1$  gets arbitrarily large, the right hand side of the equation goes to  $2 \frac{\lambda(r+\lambda)}{r}$  and the left hand side of the equation goes to zero so the equation must fail.

This equation must hold for high enough values of k, since  $\frac{e^{jk} - 1}{k}$  goes to infinity as k gets sufficiently large.

At time k, the impact on housing supply must be positive, since the housing supply is still determined by investment during the boom. The change in price when the bubble ends, denoted  $\Delta P$  is equal to  $\frac{c_1 \lambda \bar{\theta}}{(r+\lambda)c_1 + v_1}$ , which is increasing in  $\bar{\theta}$  and  $c_1$ . The cross effect is also positive.

*Proof of Proposition 4:* During a period when beliefs about the future are held constant, the equilibrium is characterized by the three equations for housing stock, investment and prices:

$$H_I(t+j) = e^{-\frac{\lambda v_1 j}{rc_1 + v_1}} H_I(t) + (1 - e^{-\frac{\lambda v_1 j}{rc_1 + v_1}}) \frac{g(t)}{(r+\lambda)v_1}, I(t+j) = \frac{\lambda e^{-\frac{\lambda v_1 j}{rc_1 + v_1}}}{rc_1 + v_1} \left( \frac{g(t)}{r+\lambda} - v_1 H_I(t) \right)$$

and  $P(t+j) = c_0 + \frac{\lambda c_1 e^{-\frac{\lambda v_1 j}{rc_1 + v_1}}}{rc_1 + v_1} \left( \frac{g(t)}{r+\lambda} - v_1 H_I(t) \right)$ . The price increase between time zero

and time one will equal  $\frac{\varepsilon \lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r+\lambda)}$ . The bubble will persist if and only if this is

greater than  $\varepsilon$  or  $\frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r+\lambda)} > 1$ . The expression  $\frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r+\lambda)}$  is monotonically increasing with  $c_1$  and equals zero when  $c_1$  equals zero. As  $c_1$  grows arbitrarily large, the expression approaches  $\frac{\lambda}{r(r+\lambda)}$ . If  $\frac{\lambda}{r(r+\lambda)} < 1$  or  $\lambda < \frac{r^2}{1-r}$ , then the bubble can never persist. If  $\frac{\lambda}{r(r+\lambda)} > 1$  or  $\lambda > \frac{r^2}{1-r}$ , then the bubble can persist if  $c_1$  is sufficiently large. If

$\lambda > \frac{r^2}{1-r}$ , then by continuity and monotonicity there exists a value of  $c_1$  where

$\frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r+\lambda)} = 1$ . Monotonicity ensures that for values of  $c_1$  above this amount, then prices will continue to rise while for prices of  $c_1$  below this amount, price growth will be lower than expectations and the bubble will start to unravel.

At the end of the second period, assuming that prices continue to rise, prices will equal

$$c_0 + \frac{(\lambda c_1)^2 e^{-\frac{2\lambda v_1}{rc_1 + v_1}} \varepsilon}{(rc_1 + v_1)^2 (r+\lambda)^2} - \frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}} (1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}}) \varepsilon}{(rc_1 + v_1)(r+\lambda)}$$

The price increase in the second period will be greater than the price increase in the first period if and only if

$$\frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r+\lambda)} + e^{-\frac{\lambda v_1}{rc_1 + v_1}} > 3$$

The left hand side of the equation is again monotonically increase in  $c_1$  and the inequality certainly fails when  $c_1$  equals zero. Assuming  $1-2r > 0$ ,

the left hand side of the equation approaches  $\frac{\lambda}{r(r+\lambda)} + 1 > 3$ , or  $\lambda > \frac{2r^2}{1-2r}$  as  $c_1$  grows.

Again, this means that there exists of value of  $c_1$  such that the inequality holds with

equality and for values of  $c_1$  above (below) this amount the inequality holds (fails)

strictly. Since  $3 - e^{-\frac{\lambda v_1}{rc_1 + v_1}}$  is greater than one, then the value of  $c_1$  that causes

$$\frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r + \lambda)} + e^{-\frac{\lambda v_1}{rc_1 + v_1}} = 3 \text{ must be higher than the value of } c_1 \text{ that causes}$$

$$\frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)(r + \lambda)} = 1.$$

*Proof of Proposition 5:* If  $c_1 < c_1^*$ , then the bubble will only last one period and the total

amount of housing built over the course of the bubble will equal  $\frac{(1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}})\varepsilon}{(r + \lambda)v_1}$ , which is

monotonically decreasing with  $c_1$ .

If  $c_1^* < c_1 < c_1^{**}$ , so that the bubble lasts exactly two periods then the amount of housing built over the course of the bubble will equal

$$\frac{e^{-\frac{\lambda v_1}{rc_1 + v_1}}(1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}})\varepsilon}{(r + \lambda)v_1} \left( \frac{(r + \lambda)(rc_1 + v_1) + \lambda c_1}{(r + \lambda)(rc_1 + v_1)} \right). \text{ The derivative of this with respect to } c_1 \text{ is}$$

$$\frac{e^{-\frac{\lambda v_1}{rc_1 + v_1}}\lambda\varepsilon}{(r + \lambda)^2(rc_1 + v_1)^2} \left( r(1 - 2e^{-\frac{\lambda v_1}{rc_1 + v_1}}) \left( r + \lambda + \frac{\lambda c_1}{(rc_1 + v_1)} \right) + 1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}} \right), \text{ which is negative if}$$

$$\text{and only if } 2e^{-\frac{\lambda v_1}{rc_1 + v_1}} > 1 + \frac{1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{r \left( r + \lambda + \frac{\lambda c_1}{(rc_1 + v_1)} \right)}.$$

If the bubble ends at period one, then prices will fall from  $c_0 + \frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{rc_1 + v_1} \frac{\varepsilon}{r + \lambda}$  to

$$c_0 - \frac{\lambda(1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}})\varepsilon}{r(r + \lambda)}, \text{ so the total price drop is } \frac{\lambda\varepsilon}{r + \lambda} \left( \frac{(2rc_1 + v_1)e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{r(rc_1 + v_1)} - \frac{1}{r} \right) \text{ which is}$$

increasing with  $c_1$ .

If the bubble ends at period two, the prices will fall from

$$c_0 + \frac{\lambda c_1 e^{-\frac{\lambda v_1}{rc_1 + v_1}}}{(rc_1 + v_1)^2 (r + \lambda)^2} \left( e^{-\frac{\lambda v_1}{rc_1 + v_1}} \lambda c_1 - (1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}})(rc_1 + v_1)(r + \lambda) \right) \text{ to}$$

$$c_0 - \frac{\lambda e^{-\frac{\lambda v_1}{rc_1 + v_1}} (1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}})\varepsilon}{r(r + \lambda)} \left( \frac{(r + \lambda)(rc_1 + v_1) + \lambda c_1}{(r + \lambda)(rc_1 + v_1)} \right). \text{ The total price drop will equal}$$

$$\frac{\lambda\varepsilon}{(r + \lambda)^2} \text{ times } \frac{e^{-2\frac{\lambda v_1}{rc_1 + v_1}} \lambda c_1^2}{(rc_1 + v_1)^2} + e^{-\frac{\lambda v_1}{rc_1 + v_1}} (1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}}) \left( \frac{v_1(r + \lambda) + \lambda c_1}{r(rc_1 + v_1)} \right). \text{ The derivative of this}$$

with respect to  $c_1$  will be positive if and only if:

$$\frac{2re^{-\frac{\lambda v_1}{rc_1 + v_1}} \lambda c_1}{(rc_1 + v_1)} + \frac{2r^2 e^{-\frac{\lambda v_1}{rc_1 + v_1}} \lambda^2 c_1^2}{(rc_1 + v_1)^2} + \lambda r \left( 1 - 2e^{-\frac{\lambda v_1}{rc_1 + v_1}} \right) \left( \frac{v_1(r + \lambda) + \lambda c_1}{(rc_1 + v_1)} \right) > \left( 1 - e^{-\frac{\lambda v_1}{rc_1 + v_1}} \right) (r(r + \lambda) - \lambda)$$

This inequality will always hold if  $r$  is sufficiently small. As  $v_1$  becomes arbitrarily large, the inequality converges to  $\lambda r(1 - 2e^{-\lambda})(r + \lambda) > (r(r + \lambda) - \lambda)(1 - e^{-\lambda})$ , which can certainly fail.